# Solving Multi-Objective Linear Fractional Stochastic Transportation Problems Involving Normal Distribution using Simulation-Based Genetic Algorithm <br> Check for updates 

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#### Abstract

In real-life situations, we human beings faced with multi-objective problems that are conflicting and non-commensurable with each other. Especially, when goods are transported from source to locations with a goal to keep exact relationships between a few parameters, those parameters of such problems might also arise in the form of fractions which are linear in nature such as; actual transportation fee/total transportation cost, delivery fee/desired path, total return/total investment, etc. Due to the uncertainty of nature, such a relationship is not deterministic. Mathematically such kinds of mathematical problems are characterized as a multi-objective linear fractional stochastic transportation problem. However, it is difficult to handle such types of mathematical problems. It can't be solved directly using mathematical programming approaches. In this paper, a solution procedure is proposed for the above problem using a stochastic Genetic Algorithm based simulation. The parameters in the constraint of the above problem follow a normal distribution. The probabilistic constraints are handled by stochastic simulation-based GA for the solution procedure of the proposed problem. The feasibility of probability constraints is checked by the stochastic programming through the Genetic Algorithm approach, without finding the equivalent deterministic model. The feasibility is maintained all-over the problem. The stochastic simulation-based Genetic Algorithm is considered to generate non-dominated solutions for the given problem. Then, a numerical case study is provided to illustrate the method.


Keywords: Genetic Algorithm, multi-objective programming, stochastic fractional programming, transportation problem.

## I. INTRODUCTION

Transportation problems with the ratio of optimization of parameters where the ratios are objective functions are known as fractional transportation problems. It is concerned with delivering the commodities from numerous assets to various locations along to keep up great connections among a

[^0]couple of parameters. Those parameters of transportation problems may happen as a proportion of actual transportation cost/total standard transportation cost, shipping cost/desired path, total return/total investment, and so forth.
In real-life, distributions of commodities are done on the minimization of the ratio of the total cost to total profit. The problem derived by such type of two linear functions gets its name as a linear fractional transportation problem (LFTP). In many real-world situations, for LFTP, decisions are often made in the presence of multiple, non-commensurable, conflicting objectives. Such kinds of problems are called multi-objective linear fractional transportation problems (MOLFTP). It deals with the distribution of goods at a time by considering the ratio of several objective functions. The parameters associated with the MOLFTP are not deterministic or fixed value always. In a mathematical programming model, uncertainties are addressed using the fuzzy program set theory or probability theory. In the present paper, we deal with the parameters to address uncertainty using probability theory. The presence of probability in a mathematical programming problem leads to a stochastic programming (SP) problem.
SP problem is one of the mathematical programming problems that involve randomness. It is concerned with the decision-making in which a few or all parameters traced as random variables for capturing uncertainty.
In our proposed work, attention has been given to solve a stochastic transportation problem having more than one linear fractional objective function. The parameters of the constraints in the above problem are normal random variables. The mathematical model is known as a multi-objective linear fractional stochastic transportation problem (MOLFSTP). However, a set of optimal solutions known as Pareto-optimal (PO) solutions occurs due to the presence of conflicting objectives in a MOLFSTP. Finding these set of PO solutions is not practically possible, rather an approximation set to the true Pareto front (PF) is expected. Researchers have attempted various methods to tackle those types of MOLFSTP problems. Nowadays, due to the popularization of the evolutionary algorithm, many researches are going on solving the above problem using the said algorithm. One such popular algorithm is the Genetic Algorithm (GA) which is an efficient algorithm for tackling such type of problems.

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Because of its population-based nature, in a single simulation run, GA can obtain multiple PO solutions. GA is superior in comparison to the classical methods. Because it finds convergent solutions, finds a diversified set of solutions, and covers the entire PF [1].
The remainder of the paper is set up as follows. Following the introduction section, the literature survey has been provided in Section 2.

Basic preliminaries are presented in Section 3. The mathematical model is defined and described in Section 4. Simulation-based GA and its solution procedure are presented in Section 5. Case study and results and discussion have been presented in Section 6 and 7, respectively. The concluding remarks are given in Section 8, followed by references.

## II. LITERATURE SURVEY

Swarup [2] was the first who proposed an LFTP. The systematic development of LFTP is found in [3, 4, 5,6,7].
An algorithm is presented by Gupta and Arora [8] to obtain the best cost-time trade-off pairs in a fractional capacitated transportation problem with bounds upon availabilities and demands. Guzel et al. [9] developed a solution procedure for fractional transportation problems with the interval coefficient. Pradhan and Biswal [10] presented a couple of algorithms to obtain an initial fundamental feasible solution of a linear fractional transportation problem.
In real-world applications, there are cases that the parameters might be inexact and have to be estimated. Due to the lack of exact data, some uncertain factors might occur within the problems. To deal with such a phenomenon,
Liu [11] found the uncertainty theory and redefined it. It was applied to address uncertain problems by many researchers to date [12, 13].
For handling uncertainty, different researchers have discussed on SP problem. Dantzig [14] was the first who formulated the SP model. Several researchers have recommended different models on SP [15, 16]. For handling uncertainty, several researchers have discussed on SP problem.

Many researchers have been developed for stochastic fractional transportation problems and their solution techniques. Charles and Dutta [17] proposed an interactive conversion technique that converts the sum of probabilistic fractional objective into the stochastic constraint with the help of a deterministic parameter. Charles and Dutta [18] applied multi-objective stochastic fractional programming problems to compiled published circuit board problems. Jain and Arya [19] presented an inverse optimization model for the transportation problem of optimizing the ratio of linear functions and linear constraints. Jadhav and Doke [20] presented a solution method to solve the fractional transportation problem wherein the coefficient of the objective function is fuzzy. Javaid, Jalil, and Asim [21] introduced a transportation problem model with a couple of fractional objectives involving random parameters.
Holland [22] developed GA, which is primarily in light of the idea of the biological process of natural selection. Holland and his understudies have devoted a great deal to the advancement of the area.
Many researchers have studied evolutionary computing and its application for solving transportation problems. GA is a
well known and effective strategy for such sort of issues. Vignaux and Michalewicz [23] discussed how to solve linear transportation problem using alternative GA. Syarif [24] developed a GA approach for solving nonlinear side constrained transportation problems. Bharathi and Vijayalakshmi [25] presented an application of evolutionary algorithms to the multi-objective transportation problem (MOTP). A solution procedure is presented for a MOTP by a fuzzy stochastic simulation-based GA by Dutta, Acharya and Mishra. [26]. A GA is applied for shipping, location, and allocation of dangerous substances to a novel bi-objective stochastic model [27]. Recently, Karthy and Ganesan [28] applied a GA for solving the MOTP. Going through the literature survey, we were motivated by a work done on MOTP using GA by Dutta, Acharya and Mishra. [26]. Their paper concentrated on fuzzy stochastic simulation-based GA as a solution procedure for a MOTP. The amount and request parameters of the restrictions follow fuzzy-exponential and fuzzy-normal distribution, respectively. However, in our proposed paper, a novel strategy has been evolved for MOLFSTP involving Normal distribution. For solving the proposed model, we implement a simulation-based GA. The main difference between this paper and the above paper is listed as follows. Firstly, this paper concentrated on a ratio of two objective functions. Secondly, both supply and demand parameters are normal random variables. Finally, there are no fuzzy parameters in this paper.

## III. BASIC PRELIMINARIES

## A. Bounded Random Number (BRN)

The function $\operatorname{rand}()$ is one way to generate random numbers between 0 and $R A N D \_M A X$, where $R A N D \_M A X$ is defined in \#stdlib as $\left(2^{10}-1\right)$ in $\mathrm{C}++$. Hence, to generate a random number in $[0,1]$, the following steps are followed.

* $m=\operatorname{rand}()$
* $m \leftarrow m / R A N D \_M A X$


## B. Normal Distribution

It is one of the probability distribution. The parameters which define Normal distribution are mean (location parameter) $\mu$ and standard deviation (scale parameter) $\sigma$.
The probability density function (pdf) of Normal distribution is defined as:

$$
\begin{equation*}
f(x)=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left\{-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right\} \tag{1}
\end{equation*}
$$

where $x>0, \mu$ is mean, $\sigma>0$ is standard deviation.
For generating Normal distribution, the following steps are used.

Step-1: Generate $m$ and $n$ from $\operatorname{BRN}(0,1)$.
Step-2: Use mean $\mu$ and standard deviation $\sigma$.
Step-3: Return $z=[-2 \ln (n)]^{1 / 2} \sin (2 \pi n)$.
Step-4: Return $\mu+\sigma z$

## C. Stochastic Simulation for Probabilistic Constraints

In a stochastic condition, some or all the coefficients of a probabilistic constraint may be random variables with a known probability distribution. For the probabilistic constraints where randomness occurs on the right-hand side are defined in (2) and (3).

$$
\begin{aligned}
& P\left(\sum_{t=1}^{n} x_{s t} \leq a_{s}\right) \geq 1-\gamma_{s} ; s=1,2, \ldots, m \\
& P\left(\sum_{s=1}^{m} x_{s t} \geq b_{t}\right) \geq 1-\delta_{t} ; t=1,2, \ldots, n
\end{aligned}
$$

(3)

Let's define $U_{s}\left(r_{1}, x\right)$ and $W_{t}\left(r_{2}, x\right)$ as follows:

$$
\begin{aligned}
& U_{s}\left(r_{1}, x\right)=\sum_{t=1}^{n} x_{s t}-a_{s} ; s=1,2, \ldots, m \\
& W_{t}\left(r_{2}, x\right)=\sum_{s=1}^{m} x_{s t}-b_{t} ; t=1,2, \ldots, n
\end{aligned}
$$

The probability constraints as defined in (2) and (3) can be written as follows:

$$
P\left(U_{s}\left(r_{1}, x\right) \leq 0\right) \geq 1-\gamma_{s} ; s=1,2, \ldots, m
$$

(4)

$$
\begin{equation*}
P\left(W_{t}\left(r_{2}, x\right) \geq 0\right) \geq 1-\delta_{t} ; t=1,2, \ldots, n \tag{5}
\end{equation*}
$$

Where $r_{1}=\left(a_{1}, a_{2}, \ldots, a_{m}\right)$ and $r_{2}=\left(b_{1}, b_{2}, \ldots, b_{n}\right)$ are an $m$-dimensional and $n$-dimensional vector of random numbers respectively, $x=\left(x_{1 t}, x_{2 t}, \ldots, x_{m t}\right) ; t=1,2, \ldots, n$ is the vector of decision variables and $\gamma_{s} ; s=1,2, \ldots, m$ and $\delta_{t} ; t=1,2, \ldots, n$ are pre specified confidence levels.
$N$ independent random vectors $r_{1}^{i}=\left(a_{1}^{i}, a_{2}^{i}, \ldots, a_{m}^{i}\right) ; i=$ $1,2, \ldots, N$ and $M$ independent random vectors $r_{2}^{j}=$ $\left(b_{1}^{j}, b_{2}^{j}, \ldots, b_{n}^{j}\right) ; j=1,2, \ldots, M$ are generated where $a_{s}^{i}$ and $b_{t}^{j}$ are random numbers generated according to the distribution of $a_{s}$ and $b_{t}$ respectively. Let $N_{s}^{\prime}(s=1,2 \ldots, m)$ and $M_{t}^{\prime}(t=1,2, \ldots, n$ be the number of cases in which $U_{s}\left(r_{1}, x\right) \leq 0 ; s=1,2, \ldots, m$ and

$$
W_{t}\left(r_{2}, x\right) \geq 0 ; t=1,2, \ldots, n \text { respectively. }
$$

Then by the definition of probability, (2) and (3) hold if $N_{s}^{\prime} /{ }_{N} \geq 1-\gamma_{s}$ and $M_{t}^{\prime} /{ }_{M} \geq 1-\delta_{t}$ respectively for $s=$ $1,2, \ldots, m ; t=1,2, \ldots, n$.

## D. Feasibility of Probability Constraints

For checking the feasibility of the probability constraints of the right hand side parameters, the following steps are used.
Step-1: Use all the steps for generating Normal distribution. i.e.,

* Generate $r_{s}, r_{s}^{\prime}, r_{t}$ and $r_{t}^{\prime}$ from $B R N(0,1)$.
$*$ Use mean $\mu_{s}, \mu_{t}^{\prime}$ and standard deviation $\sigma_{s}, \sigma_{t}^{\prime}$.
* Return $z_{s}=\left[-2 \ln \left(r_{s}^{\prime}\right)\right]^{1 / 2} \sin \left(2 \pi r_{s}\right)$ and $z_{t}=\left[-2 \ln \left(r_{t}^{\prime}\right)\right]^{1 / 2} \sin \left(2 \pi r_{t}\right)$
* Return $T_{s}=\mu_{s}+\sigma_{s} z_{s}$ and $T_{t}^{\prime}=\mu_{t}^{\prime}+\sigma_{t}^{\prime} z_{t}$

Step-2: Return $T=\sum_{s=1}^{m} T_{s}$ and $T^{\prime}=\sum_{t=1}^{n} T_{t}^{\prime}$
Step-3: Return $P=T-T^{\prime}$
Step-4: If $P \geq 0$, then the generated population is feasible for the probability constraints.

## IV. MATHEMATICAL MODEL OF MOLFSTP

Mathematically, a MOLFSTP where randomness is considered in the right-hand-side constraints is expressed as:

$$
\begin{equation*}
\operatorname{Min}: Z^{l}=\frac{\sum_{\mathrm{s}=1}^{\mathrm{m}} \sum_{\mathrm{t}=1}^{\mathrm{n}} c_{\mathrm{st}}^{1}}{\sum_{\mathrm{s}=1}^{\mathrm{m}} \sum_{\mathrm{t}=1}^{\mathrm{n}} p_{\mathrm{st}}^{1}} ; l=1,2, \ldots, L \tag{6}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& P\left(\sum_{t=1}^{n} x_{s t} \leq a_{s}\right) \geq 1-\gamma_{s} ; s=1,2, \ldots, m  \tag{7}\\
& P\left(\sum_{s=1}^{m} x_{s t} \geq b_{t}\right) \geq 1-\delta_{t} ; t=1,2, \ldots, n \tag{8}
\end{align*}
$$

where $x_{s t} \geq 0,0<\gamma_{s}, \delta_{t}<1 ; \forall s, t$.
Let $a_{s}(s=1,2, \ldots, m)$ and $b_{t}(t=1,2, \ldots, n)$ are normal random variables. The unit shipping cost coefficients along the traveled route and preferring route for transporting of goods from source $s$ to destination $t$ is represented by $c_{s t}^{l}$ and $\quad p_{s t}^{l}(s=1,2, \ldots, m ; t=1,2, \ldots, n ; l=1,2, \ldots, L)$ respectively. $\gamma_{s}$ and $\delta_{t}$ are pre specified probability levels for all $s, t$. The variable $x_{s t}$ denotes the amount transported from source $s$ to destination $t$. It is expected that the denominator of the objective function remains positive, and the total supply is greater than or equal to total demand.

## V. SIMULATION BASED GA FOR MOLFSTP

The method is designed to solve the MOLFSTP. The algorithmic steps are described as follows:

Step-1: Fix GA parameters and termination criteria (Max_gen)
Step-2: Generate the parameters for the given distribution.
Step-3: Initialize the GA population for the objective function $Z^{l}(l=1)$ with the given constraints.
Step-4: Initialize generation gen $=1$ and penalty parameter $\tau$
Step-5: Apply the bounds on the population and calculate constraints for each objective function.
Step-6: Check the feasibility condition, if satisfied go to Step 7 else go to Step 3.
Step-7: Probability criteria is checked, if satisfied go to Step 8 else go to Step 3.
Step-8: Calculate the functional value i.e., the objective function.
Step-9: Apply Selection, Crossover and Mutation respectively.
Step-10: Again, calculate the functional value i.e., the objective function.
Step-11: Again check feasibility criteria, if satisfied go to Step 12 else go to Step 3.
Step-12: Again check probability criteria, if satisfied go to Step 13 else go to Step 3.
Step-13: Apply Elitism.
Step-14: Check the stopping criteria. If reached, the current population is the best population else gen $=$ gen + 1 and go to Step 5.
Step-15: Ideal solution is obtained for the first objective function.
Step-16: Repeat the steps from Step 2 to Step 15 for the other objective function until ideal solution obtained for all objective functions.

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Step-17: Construct a pay-off matrix containing the ideal solution and functional values as shown in Table I.
Step-18: Formulate the fitness function by using bracket penalty operator:

Table I: Pay-Off Matrix

| Ideal <br> Solutions | Objective functions |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $Z^{1}(x)$ | $Z^{2}(x)$ | $\cdots$ | $Z^{L}(x)$ |
| $X^{(1)}$ | $Z^{1}\left(X^{(1)}\right)$ | $Z^{2}\left(X^{(1)}\right)$ | $\cdots$ | $Z^{L}\left(X^{(1)}\right)$ |
| $X^{(2)}$ | $Z^{1}\left(X^{(2)}\right)$ | $Z^{2}\left(X^{(2)}\right)$ | $\cdots$ | $Z^{L}\left(X^{(2)}\right)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $X^{(L)}$ | $Z^{1}\left(X^{(L)}\right)$ | $Z^{2}\left(X^{(L)}\right)$ | $\cdots$ | $Z^{L}\left(X^{(L)}\right)$ |

$$
\begin{equation*}
F^{l}(x)=Z^{l}(x)+\tau \sum_{i=1}^{s+t}<g^{i}(x)>^{2} ; l=1,2, \ldots, L \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
& Z^{l}(x)=\frac{\sum_{\mathrm{s}=1}^{\mathrm{m}} \sum_{\mathrm{t}=1}^{\mathrm{n}} c_{\mathrm{st}}^{1}}{\sum_{\mathrm{s}=1}^{\mathrm{m}} \sum_{\mathrm{t}=1}^{\mathrm{n}} p_{\mathrm{st}}^{1}} ; l=1,2, \ldots, L  \tag{10}\\
& g^{i}(x)=\sum_{t=1}^{n} x_{s t}-\left(\mu_{\mathrm{s}}+\sigma_{s} z_{s}\right) ; i=1,2, \ldots, s \\
& g^{i}(x)=\sum_{s=1}^{m} x_{s t}-\left(\mu_{t}^{\prime}+\sigma_{t}^{\prime} z_{t}\right)  \tag{11}\\
& \quad i=s+1, s+2, \ldots, s+t
\end{align*}
$$

where $x_{s t} \geq 0 ; \forall s, t . F^{l}(x)$ is a fitness function, $Z^{l}(x)$ is objective function, $\tau$ is penalty parameter, $z_{s}$ and $z_{t}$ are as described in Section $3.4,<\mathrm{g}^{i}(x)>$ is constraint violation in which $\langle\cdot\rangle$ denotes the absolute value of the operand where $<g>=\left\{\begin{array}{l}0, \mathrm{~g} \geq 0 \\ \mathrm{~g}, \mathrm{~g}<0\end{array}\right.$
Step-19: Solve using GA to obtain Pareto optimal solutions.
The algorithmic steps described earlier are displayed as a flow diagram in Fig. 1.

## VI. CASE STUDY

GAA-Oil mining"(name changed) company mines from three branches to supply the oil for five cities in India. The manager of the company decided to plan for transportation for the next month onwards. He needs to collect the primary records along with delivery capacity, demand, total profit, cost of a unit product, shipping time, and so on at the start of his project. However, due to uncertain human and natural phenomena, he can't get these data exactly. According to previous experiences, the company assumes that the supply and demand parameters follow a normal uncertain distribution with known mean and standard deviation.
The production cost and profit per unit (in liters) from source to destination are given in Table II. Similarly, the delivery time from the source to the destination for a unit is given in Table III.

Table II: Production cost/profit per unit (in rupees)

| Source | Destination |  |  |  | City 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | City 2 | City 3 | City 4 | City 5 |  |
| Branch 1 | $18 / 30$ | $17 / 32$ | $18 / 34$ | $18 / 30$ | $20 / 32$ |
| Branch 2 | $10 / 20$ | $10 / 18$ | $12 / 22$ | $9 / 20$ | $10 / 16$ |
| Branch 3 | $20 / 40$ | $18 / 32$ | $20 / 32$ | $22 / 30$ | $18 / 36$ |

Table III: Delivery time (actual/standard) per unit (in hours)

| Source | Destination |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | City 1 | City 2 | City 3 | City 4 | City 5 |
| Branch 1 | $10 / 12$ | $8 / 10$ | $10 / 13$ | $10 / 12$ | $12 / 15$ |
| Branch 2 | $6 / 8$ | $6 / 9$ | $7 / 10$ | $5 / 7$ | $6 / 9$ |
| Branch 3 | $12 / 14$ | $10 / 13$ | $12 / 15$ | $13 / 18$ | $10 / 12$ |

The mathematical model for the above problem is expressed as below in (12) to (15).
$\min : Z^{1}(x)=\frac{\sum_{s=1}^{3} \sum_{t=1}^{5} c_{s t}}{\sum_{s=1}^{3} \sum_{t=1}^{5} p_{s t}}$


Fig. 1: Flow Diagram of Simulation based GA Approach to solve MOLFSTP

$$
\begin{equation*}
Z^{2}(x)=\frac{\sum_{\mathrm{s}=1}^{3} \sum_{\mathrm{t}=1}^{5} A_{\mathrm{st}}}{\sum_{\mathrm{s}=1}^{3} \sum_{\mathrm{t}=1}^{5} S_{\mathrm{st}}} \tag{13}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& P\left(\sum_{t=1}^{5} x_{s t} \leq a_{s}\right) \geq 1-\gamma_{s} ; s=1,2,3  \tag{14}\\
& P\left(\sum_{s=1}^{3} x_{s t} \geq b_{t s}\right) \geq 1-\sigma_{t} ; t=1,2,3,4,5 \tag{15}
\end{align*}
$$

$x_{s t} \geq 0$ and $0<\gamma_{s}, \delta_{t}<1 ; \forall s, t$.
The known parameters of a normal distribution with a specified probability level (SPL) of supplies $a_{s}(s=1,2,3)$ and demands $b_{t}(1,2,3,4,5)$ are presented in Table IV and Table V respectively.

Table IV: Values of SPL; mean and standard deviation for supplies $\boldsymbol{a}_{\boldsymbol{s}}$

| Mean | standard deviation | $\operatorname{SPL}\left(\gamma_{s}\right)$ |
| :---: | :---: | :---: |
| $\mu_{1}=24$ | $\sigma_{1}=2$ | $\gamma_{1}=0.9$ |
| $\mu_{2}=32$ | $\sigma_{2}=1.5$ | $\gamma_{2}=0.9$ |
| $\mu_{3}=30$ | $\sigma_{3}=2$ | $\gamma_{3}=0.9$ |

Table V: Values of SPL; mean and standard deviation for supplies $\boldsymbol{b}_{\boldsymbol{t}}$

| Mean | standard deviation | $\operatorname{SPL}\left(\delta_{t}\right)$ |
| :---: | :---: | :---: |
| $\mu_{1}^{\prime}=12$ | $\sigma_{1}^{\prime}=1.5$ | $\delta_{1}=0.9$ |
| $\mu_{2}^{\prime}=10$ | $\sigma_{2}^{\prime}=1$ | $\delta_{2}=0.9$ |
| $\mu_{3}^{\prime}=16$ | $\sigma_{3}^{\prime}=1$ | $\delta_{3}=0.9$ |
| $\mu_{4}^{\prime}=10$ | $\sigma_{4}^{\prime}=1.5$ | $\delta_{4}=0.9$ |
| $\mu_{5}^{\prime}=14$ | $\sigma_{5}^{\prime}=1$ | $\delta_{5}=0.9$ |

Using the data in Tables IV and V, the above multi-objective fractional stochastic transportation problem is solved using simulation-based GA. Applying the steps of simulation based GA, we obtain two ideal solutions as

$$
\begin{aligned}
X^{(1)}= & (0.2541,0.3187,0.9834,0.1945,0.8094,0.1222, \\
& 0.6989,0.1251,0.5415,0.0117,0.3744,0.1642, \\
& 0.0694,0.4242,0.8113) \\
X^{(2)}= & (0.2678,0.7146,0.3646,0.1642,0.0156,0.0938 \\
& 0.2991,0.7928,0.3167,0.1896,0.0557,0.1320 \\
& 0.6188,0.7947,0.2727)
\end{aligned}
$$

The objective function values are $Z^{1}\left(X^{(1)}\right)=0.5554$ and $Z^{2}\left(X^{(2)}\right)=0.7649$ respectively.
Using the two ideal solutions a pay-off matrix is formulated in Table VI.

Table VI: Pay-Off Matrix for case study

| Ideal <br> Solutions | Objective functions |  |
| :---: | :---: | :---: |
|  | $Z^{2}$ |  |
| $X^{(1)}$ | 0.5554 | 0.7777 |
| $X^{(2)}$ | 0.5833 | 0.7649 |

Applying bracket penalty function MOLFSTP is solved as follows in (16) to (20).

$$
\begin{align*}
& F^{1}(x)=Z^{1}(x)+\tau \sum_{i=1}^{8}<g^{i}(x)>^{2}  \tag{16}\\
& F^{2}(x)=Z^{2}(x)+\tau \sum_{i=1}^{8}<g^{i}(x)>^{2} \tag{17}
\end{align*}
$$

$$
\begin{gather*}
Z^{1}(x)=\frac{\sum_{s=1}^{3} \sum_{\mathrm{t}=1}^{5} c_{s t}}{\sum_{\mathrm{s}=1}^{3} \sum_{\mathrm{t}=1}^{5} p_{s t}}  \tag{18}\\
Z^{2}(x)=\frac{\sum_{\mathrm{s}=1}^{3} \sum_{\mathrm{t}=1}^{5} A_{s t}}{\sum_{\mathrm{s}=1}^{3} \sum_{\mathrm{t}=1}^{5} S_{s t}}  \tag{19}\\
g^{i}(x)=\sum_{t=1}^{5} x_{s t}-\left(\mu_{s}+\sigma_{s} z_{s}\right) ; i=1,2,3 \\
g^{i}(x)=\sum_{s=1}^{3} x_{s t}-\left(\mu_{t}^{\prime}+\sigma_{t}^{\prime} z_{t}^{\prime}\right) ; i=4, \ldots, 8 \tag{20}
\end{gather*}
$$

## VII. RESULT AND DISCUSSION

The proposed simulation based GA approach is coded in C++ Code:: Blocks 16.01 compiler. The size of population is taken as 100 . The numbers of generations are taken to be 100 . The penalty parameter is taken for an initial value of $\tau=10$ and it is incremented by 10 after $10^{\text {th }}$ generation. The ideal solutions are recorded by taking more than 10 simulations. However, the top 4 values are record. An extensive experimental study has been done varying the value of probability of crossover $\left(P_{c}\right)$ and probability of mutation $\left(P_{m}\right)$ respectively. The $P_{c}$ has been taken $0.6,0.7,0.8$, and 0.9 and $P_{m}$ has been taken $0.001,0.005,0.01,0.05$, and 0.1 .
In these simulations, the most approximate redundant values are taken as ideal solution for each objective function. PO solutions and values are obtained and shown in Tables (VIII-XI) for different values of $P_{c}$ and $P_{m}$ respectively.
The best functional values are recurred as shown below for different values of $P_{c}$ and $P_{m}$.

Table VII: Best functional values

| $P_{m}$ | Table VII. Best |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Objective <br> function <br> value | 0.6 | 0.7 | 0.8 | 0.9 |
|  | $Z^{1}$ | 0.5461 | 0.5508 | 0.5581 | 0.5451 |
|  | $Z^{2}$ | 0.7590 | 0.7771 | 0.7716 | 0.7618 |
| 0.005 | $Z^{1}$ | 0.5547 | 0.5576 | 0.5490 | 0.5541 |
|  | $Z^{2}$ | 0.7761 | 0.7744 | 0.7610 | 0.7730 |
|  | $Z^{1}$ | 0.5512 | 0.5552 | 0.5509 | 0.5459 |
|  | $Z^{2}$ | 0.7694 | 0.7610 | 0.7718 | 0.7788 |
| 0.05 | $Z^{1}$ | 0.5521 | 0.5517 | 0.5489 | 0.5483 |
|  | $Z^{2}$ | 0.7766 | 0.7756 | 0.7662 | 0.7761 |
| 0.1 | $Z^{1}$ | 0.5498 | 0.5557 | 0.5449 | 0.5609 |
|  | $Z^{2}$ | 0.7743 | 0.7747 | 0.7763 | 0.7722 |

From the tabular results, it can be seen that for both of the objectives the best functional values are obtained at $P_{c}=0.6$ and $P_{m}=0.001$, with functional value $Z^{1}=0.5461$ and $Z^{2}=0.7590$. However, at $P_{c}=0.7$, the best functional value for the first objective is $Z^{1}=0.5508$ at $P_{m}=0.001$ and for the second objective is $Z^{2}=0.7610$ at $P_{m}=0.01$. It is also observed, at ( $P_{c}=0.8$ ) the best functional value for the first objective is $Z^{1}=0.5449$ at ( $P_{m}=0.1$ ) and for the second objective is $Z^{2}=0.7610$ at $\left(P_{m}=0.005\right)$. Lastly, at ( $P_{c}=0.9$ ) and

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( $P_{m}=0.001$ ), the best functional values for both of the objectives are $Z^{1}=0.5451$ and $Z^{2}=0.7618$ respectively. Graphical representations of the PO values are expressed in Fig. 2 to Fig. 5. From the plotted figures diversified PO values can be visualized for different values of $P_{c}$ and $P_{m}$.

## VIII. CONCLUSION

In this study, a stochastic simulation-based GA is used to solve the proposed MOLFSTP model without deriving the equivalent deterministic model. Stochastic simulation-based GA is superior in comparison to classical methods. It supports the decision-maker in forming a collection of non-dominated solutions, to obtain a diversified solution, and to cover the entire Pareto front. This also helps the decision-maker to make a more favorable choice by analyzing all the desirable way of the parameter. The non-dominated solutions represent the positions in this solution space of the problem. A numerical case study is provided to illustrate the methodology where both the supply and demand points follow a normal distribution. It is concluded from the tabular results shown in Table (VIII-XI) that the diversified Pareto optimal values can be visualized for different values of $P_{c}$ and $P_{m}$.



Fig. 4. Pareto Optimal value with $\boldsymbol{P}_{\boldsymbol{c}}=\mathbf{0 . 8}$


Fig. 5. Pareto Optimal value with $\boldsymbol{P}_{\boldsymbol{c}}=\mathbf{0 . 9}$

Fig. 2. Pareto Optimal value with $\boldsymbol{P}_{\boldsymbol{c}}=\mathbf{0 . 6}$


Fig. 3. Pareto Optimal value with $\boldsymbol{P}_{\boldsymbol{c}}=\mathbf{0 . 7}$

| Simulation | $P_{m}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | $x_{34}$ | $x_{35}$ | $Z^{1}$ | $Z^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | 0.001 | 0.2786 | 0.2786 | 0.60606 | 0.65494 | 0.52297 | 0.27077 | 0.6608 | 0.77126 | 0.81036 | 0.02835 | 0.79081 | 0.09384 | 0.04203 | 0.51222 | 0.11241 | 0.5576 | 0.77379 |
| 2nd | 0.001 | 0.33333 | 0.80938 | 0.77615 | 0.9306 | 0.22972 | 0.84262 | 0.6432 | 0.75562 | 0.75073 | 0.03519 | 0.95601 | 0.00684 | 0.82209 | 0.67058 | 0.69111 | 0.5560 | 0.79108 |
| 3rd | 0.001 | 0.26295 | 0.64321 | 0.04888 | 0.88563 | 0.06549 | 0.42815 | 0.9805 | 0.75562 | 0.65103 | 0.03617 | 0.93842 | 0.01271 | 0.42522 | 0.2131 | 0.47703 | 0.5461 | 0.79069 |
| 4th | 0.001 | 0.13978 | 0.49267 | 0.01662 | 0.60997 | 0.38416 | 0.51026 | 0.8397 | 0.84848 | 0.79961 | 0.02639 | 0.36266 | 0.24633 | 0.20528 | 0.79863 | 0.11241 | 0.57206 | 0.75902 |
| 1st | 0.005 | 0.54545 | 0.50049 | 0.39296 | 0.42913 | 0.64809 | 0.68328 | 0.4702 | 0.61486 | 0.32551 | 0.06158 | 0.93451 | 0.00195 | 0.98045 | 0.74096 | 1.0 | 0.56574 | 0.80218 |
| 2nd | 0.005 | 0.56207 | 0.84262 | 0.96872 | 0.78495 | 0.00098 | 0.6696 | 0.9062 | 0.12023 | 0.79472 | 0.01075 | 0.60899 | 0.05865 | 0.83089 | 0.84262 | 0.12023 | 0.56682 | 0.77608 |
| 3rd | 0.005 | 0.90225 | 0.71261 | 0.86021 | 0.52981 | 0.12903 | 0.23363 | 0.3314 | 0.27175 | 0.97947 | 0.12512 | 0.95797 | 0.00195 | 0.15933 | 0.63636 | 0.21799 | 0.55465 | 0.78834 |
| 4th | 0.005 | 0.83187 | 0.30303 | 0.5523 | 0.23949 | 0.47312 | 0.59433 | 0.7937 | 0.08798 | 0.71163 | 0.16325 | 0.99413 | 0.01759 | 0.5347 | 0.60313 | 0.59433 | 0.55906 | 0.79365 |
| 1st | 0.01 | 0.11144 | 0.52884 | 0.01564 | 0.2825 | 0.61779 | 0.82014 | 0.9580 | 0.81818 | 0.52297 | 0.15834 | 0.53763 | 0.00489 | 0.21896 | 0.75269 | 0.45943 | 0.5657 | 0.7694 |
| 2nd | 0.01 | 0.16325 | 0.02346 | 0.07625 | 0.69013 | 0.33333 | 0.92375 | 0.6501 | 0.21603 | 0.17498 | 0.35386 | 0.98338 | 0.00391 | 0.53079 | 0.61290 | 0.34311 | 0.56913 | 0.78661 |
| 3rd | 0.01 | 0.41349 | 0.48289 | 0.80352 | 0.98241 | 0.58455 | 0.69306 | 0.6882 | 0.02346 | 0.71847 | 0.30499 | 0.3304 | 0.01955 | 0.35484 | 0.13783 | 0.18671 | 0.55845 | 0.78336 |
| 4th | 0.01 | 0.71359 | 0.95503 | 0.75367 | 0.32063 | 0.8045 | 0.54839 | 0.84457 | 0.89834 | 0.62072 | 0.02151 | 0.33431 | 0.2043 | 0.07722 | 0.12317 | 0.31085 | 0.55157 | 0.77857 |
| 1st | 0.05 | 0.03324 | 0.05279 | 0.92082 | 0.18671 | 0.83675 | 0.02737 | 0.64712 | 0.16227 | 0.77713 | 0.20723 | 0.57576 | 0.02835 | 0.43891 | 0.13685 | 0.39394 | 0.55213 | 0.78596 |
| 2nd | 0.05 | 0.95406 | 0.19062 | 0.23851 | 0.33822 | 0.0782 | 0.54448 | 0.09873 | 0.79863 | 0.85337 | 0.00195 | 0.95503 | 0.01759 | 0.42913 | 0.82111 | 0.45748 | 0.56194 | 0.79122 |
| 3rd | 0.05 | 0.22385 | 0.43988 | 0.33529 | 0.68622 | 0.90811 | 0.82014 | 0.48876 | 0.10166 | 0.47312 | 0.07136 | 0.66862 | 0.00391 | 0.90518 | 0.04203 | 0.67253 | 0.55627 | 0.80833 |
| 4th | 0.05 | 0.84262 | 0.94135 | 0.52981 | 0.00195 | 0.25122 | 0.94526 | 0.53079 | 0.03519 | 0.98827 | 0.01662 | 0.6393 | 0.06843 | 0.73216 | 0.39198 | 0.02933 | 0.55355 | 0.78025 |
| 1st | 0.1 | 0.02151 | 0.20235 | 0.67742 | 0.14956 | 0.12317 | 0.79277 | 0.4741 | 0.44184 | 0.80254 | 0.00293 | 0.59531 | 0.24438 | 0.52004 | 0.5347 | 0.35875 | 0.55009 | 0.77544 |
| 2nd | 0.1 | 0.478 | 0.4868 | 0.95406 | 0.3001 | 0.70577 | 0.51026 | 0.99609 | 0.06354 | 0.31476 | 0.00587 | 0.75562 | 0.00489 | 0.04594 | 0.70283 | 0.06158 | 0.56739 | 0.77443 |
| $3^{\text {rd }}$ | 0.1 | 0.38514 | 0.348 | 0.97361 | 0.48485 | 0.89052 | 0.75171 | 0.69795 | 0.59726 | 0.64614 | 0.13099 | 0.52884 | 0.02444 | 0.71261 | 0.23754 | 0.00489 | 0.56275 | 0.77425 |
| 4th | 0.1 | 0.09189 | 0.80645 | 0.77615 | 0.16715 | 0.14663 | 0.41251 | 0.11241 | 0.23265 | 0.53763 | 0.03324 | 0.08993 | 0.00684 | 0.56403 | 0.10362 | 0.01466 | 0.54977 | 0.77503 |

Table VIII: Pareto optimal solutions for $\boldsymbol{P}_{\boldsymbol{c}}=0.6$

| Simulation | $P_{m}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{15}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | $x_{34}$ | $x_{35}$ | $Z^{1}$ | $Z^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | 0.001 | 0.4956 | 0.08113 | 0.85435 | 0.89736 | 0.42424 | 0.17595 | 0.79081 | 0.31378 | 0.12415 | 0.7175 | 0.02346 | 0.79179 | 0.05376 | 0.44184 | 0.56083 | 0.79967 |
| 2nd | 0.001 | 0.75758 | 0.07038 | 0.92375 | 0.80156 | 0.44477 | 0.68035 | 0.62561 | 0.07527 | 0.30401 | 0.11437 | 0.02151 | 0.33333 | 0.87879 | 0.01564 | 0.59561 | 0.76178 |
| $3^{\text {rd }}$ | 0.001 | 0.8739 | 0.83773 | 0.69795 | 0.12903 | 0.39687 | 0.17107 | 0.14467 | 0.67351 | 0.10753 | 0.89443 | 0.04399 | 0.29912 | 0.00098 | 0.23558 | 0.5451 | 0.80809 |
| 4th | 0.001 | 0.54839 | 0.66276 | 0.32747 | 0.18084 | 0.27761 | 0.72923 | 0.12512 | 0.3216 | 0.14858 | 0.20723 | 0.0391 | 0.79961 | 0.25415 | 0.53275 | 0.56342 | 0.79144 |
| 1st | 0.005 | 0.55327 | 0.26197 | 0.62268 | 0.78006 | 0.42033 | 0.55132 | 0.14858 | 0.07429 | 0.18866 | 0.913 | 0.03226 | 0.12219 | 0.22092 | 0.652 | 0.55414 | 0.80777 |
| 2nd | 0.005 | 0.54839 | 0.25122 | 0.28446 | 0.65494 | 0.64516 | 0.99218 | 0.17498 | 0.23949 | 0 | 0.34213 | 0.02346 | 0.14858 | 0.6129 | 0.19648 | 0.5781 | 0.77301 |
| 3rd | 0.005 | 0.91789 | 0.34213 | 0.7957 | 0.34897 | 0.86901 | 0.28055 | 0.63441 | 0.81623 | 0.00391 | 0.3998 | 0.02835 | 0.32258 | 0.37732 | 0.03226 | 0.5589 | 0.77698 |
| 4th | 0.005 | 0.89345 | 0.66178 | 0.17595 | 0.58162 | 0.90616 | 0.26979 | 0.83285 | 0.7781 | 0.02737 | 0.77224 | 0.32845 | 0.54937 | 0.2131 | 0.18768 | 0.55481 | 0.78807 |
| 1st | 0.01 | 0.26393 | 0.60997 | 0.24927 | 0.23558 | 0.86119 | 0.4565 | 0.07625 | 0.32356 | 0.05376 | 0.5523 | 0.00489 | 0.37243 | 0.94721 | 0.11339 | 0.58041 | 0.77998 |
| 2nd | 0.01 | 0.16813 | 0.88661 | 0.68328 | 0.89541 | 0.87977 | 0.91984 | 0.15543 | 0.61779 | 0.38319 | 0.8436 | 0.02737 | 0.04008 | 0.24731 | 0.36168 | 0.54593 | 0.78084 |
| 3rd | 0.01 | 0.76637 | 0.36266 | 0.80352 | 0.17595 | 0.04008 | 0.69208 | 0.99023 | 0.81036 | 0.03421 | 0.56207 | 0.02933 | 0.03323 | 0.92082 | 0.6002 | 0.5638 | 0.77884 |
| 4th | 0.01 | 0.70381 | 0.57771 | 0.64809 | 0.15249 | 0.739 | 0.56403 | 0.90518 | 0.72336 | 0.01271 | 0.53666 | 0.00782 | 0.77517 | 0.38319 | 0.23069 | 0.55829 | 0.7797 |
| 1st | 0.05 | 0.2219 | 0.94037 | 0.77126 | 0.03226 | 0.85826 | 0.20723 | 0.71359 | 0.19844 | 0.00098 | 0.26295 | 0.10753 | 0.95601 | 0.81916 | 0.20821 | 0.57445 | 0.77665 |
| 2nd | 0.05 | 0.52981 | 0.59042 | 0.8348 | 0.00098 | 0.6999 | 0.34702 | 0.37537 | 0.56207 | 0.00195 | 0.28641 | 0.11144 | 0.39003 | 0.59433 | 0.72141 | 0.55253 | 0.78943 |
| 3rd | 0.05 | 0.83382 | 0.30499 | 0.14272 | 0.26295 | 0.3998 | 0.4565 | 0.97165 | 0.72532 | 0.13881 | 0.04008 | 0.00098 | 0.49365 | 0.68231 | 0.46921 | 0.57713 | 0.77613 |
| 4th | 0.05 | 0.10753 | 0.82209 | 0.59726 | 0.57771 | 0.23656 | 0.75073 | 0.93744 | 0.96188 | 0.01369 | 0.44086 | 0.00489 | 0.97752 | 0.04985 | 0.75758 | 0.54832 | 0.79169 |
| 1st | 0.1 | 0.26491 | 0.27468 | 0.49365 | 0.56305 | 0.78397 | 0.39003 | 0.61975 | 0.01662 | 0.0303 | 0.88759 | 0.03519 | 0.81525 | 0.5044 | 0.97263 | 0.5609 | 0.80671 |
| 2nd | 0.1 | 0.11535 | 0.18084 | 0.52102 | 0.17693 | 0.54057 | 0.20919 | 0.68622 | 0.4174 | 0.0088 | 0.14272 | 0.04594 | 0.13587 | 0.50342 | 0.41251 | 0.56308 | 0.77921 |
| 3rd | 0.1 | 0.26784 | 0.21896 | 0.82502 | 0.81036 | 0.93451 | 0.77322 | 0.99805 | 0.11633 | 0.07527 | 0.27859 | 0.00977 | 0.29912 | 0.38416 | 0.31378 | 0.56655 | 0.77309 |
| 4th | 0.1 | 0.05865 | 0.78006 | 0.87097 | 0.56305 | 0.76735 | 0.29521 | 0.41153 | 0.52981 | 0.00293 | 0.60704 | 0.02151 | 0.79961 | 0.56501 | 0.06549 | 0.56621 | 0.78128 |

Table IX: Pareto optimal solutions for $\boldsymbol{P}_{\boldsymbol{c}}=\mathbf{0 . 7}$

## Solving MOLFSTP Involving Normal Distribution Using Simulation Based GA

| Simulation | $P_{m}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{15}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | $x_{34}$ | $x_{35}$ | $Z^{1}$ | $Z^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | 0.001 | 0.4956 | 0.08113 | 0.85435 | 0.89736 | 0.42424 | 0.17595 | 0.79081 | 0.31378 | 0.12415 | 0.7175 | 0.02346 | 0.79179 | 0.05376 | 0.44184 | 0.56083 | 0.79967 |
| 2nd | 0.001 | 0.75758 | 0.07038 | 0.92375 | 0.80156 | 0.44477 | 0.68035 | 0.62561 | 0.07527 | 0.30401 | 0.11437 | 0.02151 | 0.33333 | 0.87879 | 0.01564 | 0.59561 | 0.76178 |
| $3^{\text {rd }}$ | 0.001 | 0.8739 | 0.83773 | 0.69795 | 0.12903 | 0.39687 | 0.17107 | 0.14467 | 0.67351 | 0.10753 | 0.89443 | 0.04399 | 0.29912 | 0.00098 | 0.23558 | 0.5451 | 0.80809 |
| 4th | 0.001 | 0.54839 | 0.66276 | 0.32747 | 0.18084 | 0.27761 | 0.72923 | 0.12512 | 0.3216 | 0.14858 | 0.20723 | 0.0391 | 0.79961 | 0.25415 | 0.53275 | 0.56342 | 0.79144 |
| 1st | 0.005 | 0.55327 | 0.26197 | 0.62268 | 0.78006 | 0.42033 | 0.55132 | 0.14858 | 0.07429 | 0.18866 | 0.913 | 0.03226 | 0.12219 | 0.22092 | 0.652 | 0.55414 | 0.80777 |
| 2nd | 0.005 | 0.54839 | 0.25122 | 0.28446 | 0.65494 | 0.64516 | 0.99218 | 0.17498 | 0.23949 | 0 | 0.34213 | 0.02346 | 0.14858 | 0.6129 | 0.19648 | 0.5781 | 0.77301 |
| 3rd | 0.005 | 0.91789 | 0.34213 | 0.7957 | 0.34897 | 0.86901 | 0.28055 | 0.63441 | 0.81623 | 0.00391 | 0.3998 | 0.02835 | 0.32258 | 0.37732 | 0.03226 | 0.5589 | 0.77698 |
| 4th | 0.005 | 0.89345 | 0.66178 | 0.17595 | 0.58162 | 0.90616 | 0.26979 | 0.83285 | 0.7781 | 0.02737 | 0.77224 | 0.32845 | 0.54937 | 0.2131 | 0.18768 | 0.55481 | 0.78807 |
| 1st | 0.01 | 0.26393 | 0.60997 | 0.24927 | 0.23558 | 0.86119 | 0.4565 | 0.07625 | 0.32356 | 0.05376 | 0.5523 | 0.00489 | 0.37243 | 0.94721 | 0.11339 | 0.58041 | 0.77998 |
| 2nd | 0.01 | 0.16813 | 0.88661 | 0.68328 | 0.89541 | 0.87977 | 0.91984 | 0.15543 | 0.61779 | 0.38319 | 0.8436 | 0.02737 | 0.04008 | 0.24731 | 0.36168 | 0.54593 | 0.78084 |
| 3rd | 0.01 | 0.76637 | 0.36266 | 0.80352 | 0.17595 | 0.04008 | 0.69208 | 0.99023 | 0.81036 | 0.03421 | 0.56207 | 0.02933 | 0.03323 | 0.92082 | 0.6002 | 0.5638 | 0.77884 |
| 4th | 0.01 | 0.70381 | 0.57771 | 0.64809 | 0.15249 | 0.739 | 0.56403 | 0.90518 | 0.72336 | 0.01271 | 0.53666 | 0.00782 | 0.77517 | 0.38319 | 0.23069 | 0.55829 | 0.7797 |
| 1st | 0.05 | 0.2219 | 0.94037 | 0.77126 | 0.03226 | 0.85826 | 0.20723 | 0.71359 | 0.19844 | 0.00098 | 0.26295 | 0.10753 | 0.95601 | 0.81916 | 0.20821 | 0.57445 | 0.77665 |
| 2nd | 0.05 | 0.52981 | 0.59042 | 0.8348 | 0.00098 | 0.6999 | 0.34702 | 0.37537 | 0.56207 | 0.00195 | 0.28641 | 0.11144 | 0.39003 | 0.59433 | 0.72141 | 0.55253 | 0.78943 |
| 3rd | 0.05 | 0.83382 | 0.30499 | 0.14272 | 0.26295 | 0.3998 | 0.4565 | 0.97165 | 0.72532 | 0.13881 | 0.04008 | 0.00098 | 0.49365 | 0.68231 | 0.46921 | 0.57713 | 0.77613 |
| 4th | 0.05 | 0.10753 | 0.82209 | 0.59726 | 0.57771 | 0.23656 | 0.75073 | 0.93744 | 0.96188 | 0.01369 | 0.44086 | 0.00489 | 0.97752 | 0.04985 | 0.75758 | 0.54832 | 0.79169 |
| 1st | 0.1 | 0.26491 | 0.27468 | 0.49365 | 0.56305 | 0.78397 | 0.39003 | 0.61975 | 0.01662 | 0.0303 | 0.88759 | 0.03519 | 0.81525 | 0.5044 | 0.97263 | 0.5609 | 0.80671 |
| 2nd | 0.1 | 0.11535 | 0.18084 | 0.52102 | 0.17693 | 0.54057 | 0.20919 | 0.68622 | 0.4174 | 0.0088 | 0.14272 | 0.04594 | 0.13587 | 0.50342 | 0.41251 | 0.56308 | 0.77921 |
| 3rd | 0.1 | 0.26784 | 0.21896 | 0.82502 | 0.81036 | 0.93451 | 0.77322 | 0.99805 | 0.11633 | 0.07527 | 0.27859 | 0.00977 | 0.29912 | 0.38416 | 0.31378 | 0.56655 | 0.77309 |
| 4th | 0.1 | 0.05865 | 0.78006 | 0.87097 | 0.56305 | 0.76735 | 0.29521 | 0.41153 | 0.52981 | 0.00293 | 0.60704 | 0.02151 | 0.79961 | 0.56501 | 0.06549 | 0.56621 | 0.78128 |

Table X: Pareto optimal for $\boldsymbol{P}_{\boldsymbol{c}}=0.8$

| Simulation | $P_{m}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | $x_{34}$ | $x_{35}$ | $Z^{1}$ | $Z^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | 0.001 | 0.4956 | 0.08113 | 0.85435 | 0.62952 | 0.89736 | 0.42424 | 0.17595 | 0.79081 | 0.31378 | 0.12415 | 0.7175 | 0.02346 | 0.79179 | 0.05376 | 0.44184 | 0.56083 | 0.79967 |
| 2nd | 0.001 | 0.75758 | 0.07038 | 0.92375 | 0.36364 | 0.80156 | 0.44477 | 0.68035 | 0.62561 | 0.07527 | 0.30401 | 0.11437 | 0.02151 | 0.33333 | 0.87879 | 0.01564 | 0.59561 | 0.76178 |
| 3 dd | 0.001 | 0.8739 | 0.83773 | 0.69795 | 0.99805 | 0.12903 | 0.39687 | 0.17107 | 0.14467 | 0.67351 | 0.10753 | 0.89443 | 0.04399 | 0.29912 | 0.00098 | 0.23558 | 0.5451 | 0.80809 |
| 4th | 0.001 | 0.54839 | 0.66276 | 0.32747 | 0.0567 | 0.18084 | 0.27761 | 0.72923 | 0.12512 | 0.3216 | 0.14858 | 0.20723 | 0.0391 | 0.79961 | 0.25415 | 0.53275 | 0.56342 | 0.79144 |
| 1st | 0.005 | 0.55327 | 0.26197 | 0.62268 | 0.26979 | 0.78006 | 0.42033 | 0.55132 | 0.14858 | 0.07429 | 0.18866 | 0.913 | 0.03226 | 0.12219 | 0.22092 | 0.652 | 0.55414 | 0.80777 |
| 2nd | 0.005 | 0.54839 | 0.25122 | 0.28446 | 0.31867 | 0.65494 | 0.64516 | 0.99218 | 0.17498 | 0.23949 | 0 | 0.34213 | 0.02346 | 0.14858 | 0.6129 | 0.19648 | 0.5781 | 0.77301 |
| 3rd | 0.005 | 0.91789 | 0.34213 | 0.7957 | 0.47214 | 0.34897 | 0.86901 | 0.28055 | 0.63441 | 0.81623 | 0.00391 | 0.3998 | 0.02835 | 0.32258 | 0.37732 | 0.03226 | 0.5589 | 0.77698 |
| 4th | 0.005 | 0.89345 | 0.66178 | 0.17595 | 0.37048 | 0.58162 | 0.90616 | 0.26979 | 0.83285 | 0.7781 | 0.02737 | 0.77224 | 0.32845 | 0.54937 | 0.2131 | 0.18768 | 0.55481 | 0.78807 |
| 1 st | 0.01 | 0.26393 | 0.60997 | 0.24927 | 0.99511 | 0.23558 | 0.86119 | 0.4565 | 0.07625 | 0.32356 | 0.05376 | 0.5523 | 0.00489 | 0.37243 | 0.94721 | 0.11339 | 0.58041 | 0.77998 |
| 2nd | 0.01 | 0.16813 | 0.88661 | 0.68328 | 0.15738 | 0.89541 | 0.87977 | 0.91984 | 0.15543 | 0.61779 | 0.38319 | 0.8436 | 0.02737 | 0.04008 | 0.24731 | 0.36168 | 0.54593 | 0.78084 |
| 3rd | 0.01 | 0.76637 | 0.36266 | 0.80352 | 0.52590 | 0.17595 | 0.04008 | 0.69208 | 0.99023 | 0.81036 | 0.03421 | 0.56207 | 0.02933 | 0.03323 | 0.92082 | 0.6002 | 0.5638 | 0.77884 |
| 4th | 0.01 | 0.70381 | 0.57771 | 0.64809 | 0.55621 | 0.15249 | 0.739 | 0.56403 | 0.90518 | 0.72336 | 0.01271 | 0.53666 | 0.00782 | 0.77517 | 0.38319 | 0.23069 | 0.55829 | 0.7797 |
| 1st | 0.05 | 0.2219 | 0.94037 | 0.77126 | 0.63441 | 0.03226 | 0.85826 | 0.20723 | 0.71359 | 0.19844 | 0.00098 | 0.26295 | 0.10753 | 0.95601 | 0.81916 | 0.20821 | 0.57445 | 0.77665 |
| 2nd | 0.05 | 0.52981 | 0.59042 | 0.8348 | 0.02053 | 0.00098 | 0.6999 | 0.34702 | 0.37537 | 0.56207 | 0.00195 | 0.28641 | 0.11144 | 0.39003 | 0.59433 | 0.72141 | 0.55253 | 0.78943 |
| 3rd | 0.05 | 0.83382 | 0.30499 | 0.14272 | 0.56305 | 0.26295 | 0.3998 | 0.4565 | 0.97165 | 0.72532 | 0.13881 | 0.04008 | 0.00098 | 0.49365 | 0.68231 | 0.46921 | 0.57713 | 0.77613 |
| 4th | 0.05 | 0.10753 | 0.82209 | 0.59726 | 0.52004 | 0.57771 | 0.23656 | 0.75073 | 0.93744 | 0.96188 | 0.01369 | 0.44086 | 0.00489 | 0.97752 | 0.04985 | 0.75758 | 0.54832 | 0.79169 |
| 1st | 0.1 | 0.26491 | 0.27468 | 0.49365 | 0.46334 | 0.56305 | 0.78397 | 0.39003 | 0.61975 | 0.01662 | 0.0303 | 0.88759 | 0.03519 | 0.81525 | 0.5044 | 0.97263 | 0.5609 | 0.80671 |
| 2nd | 0.1 | 0.11535 | 0.18084 | 0.52102 | 0.60606 | 0.17693 | 0.54057 | 0.20919 | 0.68622 | 0.4174 | 0.0088 | 0.14272 | 0.04594 | 0.13587 | 0.50342 | 0.41251 | 0.56308 | 0.77921 |
| 3rd | 0.1 | 0.26784 | 0.21896 | 0.82502 | 0.57576 | 0.81036 | 0.93451 | 0.77322 | 0.99805 | 0.11633 | 0.07527 | 0.27859 | 0.00977 | 0.29912 | 0.38416 | 0.31378 | 0.56655 | 0.77309 |
| 4th | 0.1 | 0.05865 | 0.78006 | 0.87097 | 0.75171 | 0.56305 | 0.76735 | 0.29521 | 0.41153 | 0.52981 | 0.00293 | 0.60704 | 0.02151 | 0.79961 | 0.56501 | 0.06549 | 0.56621 | 0.78128 |

Table XI: Pareto optimal solutions for $P_{c}=0.9$

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