

# Identification Features of External and Internal Variables in the Mathematical Model of Educational Trajectory



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**Abstract:** *The article describes the main stages of identification and formation of variables which are included in the developed mathematical model of the student's educational trajectory. Analyzed are such groups of external variables as geographical location, age and gender, learning style and parameters of user interaction with the user interface, academic performance, level of possession of complex skills. The numerical values of the variables in different conditions are determined. The types of sessions in which information is collected are highlighted. Data arrays from such information sources as the GlobalLab online platform and the electronic diary Diary.ru were analyzed. Using the studied variables, a mathematical model of the student is presented, which takes into account many properties. The aim of the project is to create a technological model for the application of machine learning methods to predict the optimal educational trajectory of the student. Achieving this goal and using the scientific and technical results of the Project will provide a number of useful technical, technological and technical and economic effects.*

**Keywords:** *mathematical model, machine learning, online training, GlobalLab, educational trajectory*

## I. INTRODUCTION

The use of machine learning methods for the formation of personal educational paths through full or partial automation will significantly reduce the cost of individualization of training, which, as many domestic and foreign pedagogical studies show, has a positive effect on the level of educational results.

### The development of skills

Nowadays, teachers need the minimum skills to incorporate technology into teaching: They must have at least a minimum of hardware and software skills and equipment in an advanced engineering school. In addition to the minimum number, teachers need a full range of support, such as design and planning, instructional design, the equipment or equipment listed above, or complete skills that often require an unreasonable amount of time, money and effort to gain

time, money and effort. they further aggravate other professional activities that are rewarded in the job and promotion process. The types of support that CU-Boulder should provide for the effective and widespread use of training techniques include:

- Planning instructions;
- Media production;
- Learning management systems;
- Facilities and equipment.

Support can take many forms: traditional support or desk, short-term or long-term counseling and training. Many campus units currently provide one or more types of support, and they do not understand what the unit does, for whom, and with what efficiency. Support and training for all training technologies should be closely integrated into existing teacher and assistant development programs. The campus should continue to provide educational technology when the faculty or department wants the service unit to perform its function by supporting media education or research [13]. Most teachers use technology to improve or facilitate the organization and management of courses by posting training programs on the course website or by displaying Power Point slides instead of overhead and transparent materials. There is little or no change in this way of use in educational practice, and they are often translated into new media with greater efficiency. Faculties receive different support services for different technologies: production and design support for basic website development, equipment support and classroom equipment. Support for learning management systems also facilitates the use of this educational technology. Some teachers use technology in ways that require strong design support. In other words, they are changing teaching methods to fit existing and emerging technologies. For example, a teacher may create an interactive three-dimensional Java program to demonstrate it scientifically.

## II. METHODOLOGY

Arrays of data from the GlobalLab online platform and the electronic diary Diary.ru were used to form a model of the student's educational trajectory. The methodology of mathematical modeling, analysis and selection of variables was used. In the work, types of sessions were formed to collect information about students, and for each type of sessions, the values of variables averaged over all sessions were clustered using the k-means method.

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For mathematical modeling, Jupyter Notebook, GNU Octave software was used.

By configuring machine learning models to take into account each participant profiles, recommendations can be made to create the most efficient teams and hence produce the best results.

When working on literature, the key was to explore the possibility of developing online math courses in line with the ideas presented in the introduction.

Literature can be divided into the following sections:

- 1) research into the technique of identifying learning;
- 2) research that explores methods for creating personalized training practices for online courses;
- 3) studies looking at the aspect of individualization in the context of specific features of mental health processes;
- 4) studies on the social and humanities of online education;
- 5) a study on the technical aspects of creating and delivering online courses [11, 12].

## III. RESULT AND DISCUSSION

At Stage 1 of the applied scientific research (ASR), all the variables used in mathematical modeling of both the subject and the object of the educational path were proposed to be divided into two main clusters:

- 1) a cluster of external variables;
- 2) a cluster of internal variables.

External variables include variables that describe the student's properties, independent of the Stage within the educational path. Such properties, in particular, include gender, learning style, student age. External variables and methods for calculating their values are a mathematical model of a student as a subject of an educational trajectory. In this regard, at Stage 2 of the ASR, as part of the solution to the problem of developing a mathematical model of a student, researchers needed to:

- 1) Consider a list of hypothetical external variables whose values can be obtained from data sources available to researchers (data from the GlobalLab platform, data from the electronic diary Diary.ru).
- 2) Determine the number of students for whom data are available for each hypothetical external variable.
- 3) Determine the possibility and method of restoring or modeling the values of a hypothetical external variable if its value cannot be obtained directly from data sources.
- 4) Make a decision to include a hypothetical external variable in the final list of variables included in the student's mathematical model.
- 5) If you include a variable in the final list, set the method for calculating the value of the variable based on the source data.

At Stage 1 of the ASR, the groups presented in Table 1 were included in the cluster of external variables [1].

**Table 1. Groups of external variables highlighted in Stage 1**

No.	Group of variables	Planned number of variables in the group
1	Geographical position	2-3
2	Age and gender	2
3	Learning style and user interaction parameters	4-6

4	Academic performance (grades, test results)	1-5
5	Level of proficiency in complex skills	1-5

At Stage 2 of ASR, hypothetical variables of all 5 groups were analyzed. Below is the progress and results of the analysis of each of the groups.

To give a mathematical model of an educational trajectory of a finished form, the concept of time must be introduced into it. The trajectory consists of events (states), not just following each other, but also having two very important temporal characteristics: duration and interval.

Duration indicates how long a particular state of the trajectory lasted, while the interval reflects how far this state is from the previous one. In order to unify these two characteristics, it was proposed to represent the period of the absence of any event in the trajectory as an event of a special type - an inactivity event. Thus, the duration becomes another characteristic of the state of the trajectory, and the interval is such a characteristic for the inactivity event.

In accordance with given the representation of the events of the educational trajectory, the hypothesis takes the form of (1, 2):

$$x_t = \{e_t, d_t\} \quad (1)$$

$$h_t = f(ht_{-1}, x_t; \theta) \quad (2)$$

where  $d_t$  is the duration of the condition described by  $e_t$ .

To include the characteristics of the continuing time  $d_t$  in the mathematical model of the educational trajectory, an event contextualization method was proposed.

In the framework of modeling the characteristics of the duration of the state of the educational trajectory, the vector of the temporary context  $c^d$  is calculated first (3):

$$c^d = \varphi(\log(d_t); \theta) \quad (3)$$

where  $\varphi$  is the nonlinear transformation  $dt$  performed by the direct propagation method in a neural network with a set of parameters  $\theta$ .

The logarithmic transformation  $d_t$  is performed in order to smooth out a wide range of values expressing time intervals and to make possible the simultaneous use of periods expressed in days, months, and even years [9].

The calculation of the time mask vector is carried out by linear transformation  $c^d$  using a set of weights  $W_d \in R^{C \times E}$  and offsets  $b_d \in R^E$ . The result of the transformation is then transferred to the sigmoidal nonlinear activation function  $\sigma$  to obtain the mask  $m_d \in R^E$  and  $R^E \rightarrow [0; 1]$  (4).  $C$  is the dimension of the vector of the temporary context,  $E$  is the dimension of the vector of embedding the state of the educational trajectory.

$$m_d = \sigma(c^d W_d + b_d) \quad (4)$$

The resulting vector of the temporary mask is then superimposed on the vector of embedding the state of the educational trajectory by applying the operation of the element-wise product (5):

$$q_t \leftarrow x_t \odot m_d \quad (5)$$

The resulting vector, in turn, is input to the recursive layer of the RNS. Thus, the vector  $q_t$ , which simulates one Stage of the educational path, is used as input for the mathematical model of using the xMANN recurrence network developed in Stage 1 of the ASR [1].



Since the input variables of the model of the educational trajectory change over time, it can be characterized as dynamic. Due to the use of nonlinear transformations in presenting the characteristics of the duration of the state of the educational trajectory (4) and the sigmoidal function of nonlinear activation, the calculation of the time mask vector (5) is a non-linear model of the educational trajectory. When calculating the time mask vector, a set of weights  $W_d$  is applied, which are initialized by random variables, which determines the stochastic nature of the model.

According to the way of representing the object of modeling, the model of the educational trajectory is functional for the reasons already described above in relation to the student's model.

At Stage 2 of the ASR, a final list of events is formed, which are taken into account by the mathematical model of the educational trajectory. The list of events allows us to increase the adequacy of the mathematical model of the educational trajectory by including in it a wide range of educational activities that are not traditionally included in student models [4-7].

These assessments can offer potential educational decisions on the quality of learning outcomes and ensure equity in the allocation of learning opportunities. Thus, learning the trajectory of learning experiences would support policy goals, set measurable goals, assist and even reform the appropriate education system. In addition, benchmarking will help countries to better understand the risks and challenges of student learning, to explore their relative strengths and weaknesses, and to monitor progress.

This facilitates the ability to correlate outcomes with learning goals, in addition to the proposed innovative approach that reflects students' abilities and considers how these skills relate to adulthood [2].

Defining the path of learning and learning, van den Hevel-Panhuizen (2008) notes that there are three intertwined meanings: a learning trajectory that gives an overview of the student learning process and a learning trajectory consisting of didactic features that describe how it can best integrate and encourage the learning process and syllabus, indicating which of the major elements of the mathematics curriculum should be taught.

Van den Heuvel-Panhuizen describes how the "learning-to-learn" (TAL8) trajectory, with transitional goals in primary schools, is based on early numerical experiences for children. They represent TAL numbers for the youngest children on three levels. The first level is called the "Emergency number" (pre-school year), the second is the "Increasing sense of number" (kindergarten 1 and 2), and the third - "Calculations up to 20" (1st and 2nd grade) [14, 17]. Further discussion of the importance of this work, as it relates to numbers, geometry and dimensions. The intention is to extend this work to secondary education. The learning and learning trajectory are seen as part of the "longitudinal perspective" all teachers must adhere to. It is seen that it goes beyond the textbook and beyond the tests, but focuses on achievement goals and the overall learning activity that can contribute to their achievement. In particular, "level" understanding of domains is considered as potentially useful for indicating utility paths, i.e. [15, 18, 19].

Those relate to particular school years or grades. They are

also seen as useful in raising the level that is, directing children towards the final basic goals of basic mathematics education. It is also suggested that publicly available curricula offer teachers a way to control children's development [3].

#### AI projections on the path of student learning

Artificial intelligence in schools will enable students to analyze students' abilities and performance and point ahead with predictive analytics [8].

Some developers are currently developing machine learning prediction models to achieve this. For example, if student letter scores are constantly declining, the program can predict how it will behave in the future if the student does not take appropriate action. Then the teacher can look for other ways of adapting, directing, or helping students better [13, 16, 20].

**Table 2. Qualities of learning trajectories (left) and mis-perceptions (right) [10]**

What Learning Trajectories Are	What Learning Trajectories Are Not
Domain-specific models	General or universal principles
Expected probabilities	Stage theories
Empirically-based models of student thinking	Logico-mathematical deconstructions
Based in students' thinking	Based in opinions of experts in mathematics
Elicited by rich or novel tasks	Derived from typical exercises
Include strategies, reasons, explanations and cases	Sub-goals of the target Include exploring misconceptions
A means to avoid errors	
Ordered by increasing sophistication	Ordered by difficulty
Connected to big ideas over the long term	Curriculum material
Evolving	Fixed

#### IV. CONCLUSION

At Stage 2 ASR, a mathematical model of the educational trajectory is developed, which includes:

- external variables that reflect the student's properties, including student learning outcomes;
- internal variables that reflect the basic properties of the Stage of the educational trajectory at the time of each event in it;
- temporal characteristics expressing the duration and intervals of Stages that make up the educational trajectory, and thus reflecting their structure.

Based on the results of work, it can be concluded that the mathematical model of the student (subject of the educational trajectory) was successfully formed, which included a total of 43 variables that reflect the following basic properties of students:

- geographical location, including the accessibility of cultural and educational facilities and indirect reflection of climatic conditions (3 variables);
- gender and age (3 variables);
- parameters of user interaction with the user interface (25 variables);
- the style of his training (4 variables);



- academic results, including weighted average grades in subjects of various profiles and test results taking into account the time taken to complete the assignment (6 variables);
- the development of complex skills of both the student himself and, in some cases, his teacher (2 variables).

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