Inventory System with Shortages and Weibull Deterioration Purpose

Kapil Kumar Bansal, Ompal Singh, Satish Kumar, Priti Chaudhary

***Abstract:** In the paper, a venture has been made to develop a stock presentation for interminable planning horizon with exponentially growing interest value. It might be seen that debilitating doesn't depend on time as it were. It can impact as a result of climate conditions, clamminess, and capacity conditions, etc in this way it is progressively reasonable to consider rot rate as two-parameter Weibull spread work. Inadequacy is allowed and totally multiplied. The holding cost contemplated a direct limit of time. The ideal solution of the proposed stock show is construed and pondered same cases.*

***Keywords:** Inventory System, Deterioration, Weibull distribution.*

I. INTRODUCTION

The two-warehouse model can be connected to numerous down to earth circumstances, because of the presence of open market strategy; the business rivalry turns out to be extremely high to involve the most extreme conceivable market. Subsequently, the administration of the departmental store is limited to contract a different warehouse on rental premise at a separation place for putting away of abundance things. Complete accumulated deficiencies are allowed in this investigation.

The models are quite useful in practice. In the previously, many authors have considered inventory models for diminishing items stored in two warehouses. Sarma (1987) A deterministic inventory model for deteriorating items with two storage facilities. Pakkala and Achary (1992a) considered discrete time inventory model for deteriorating items with two warehouses. Yang (2006) studied model with inflation in which shortages are considered to be partially backlogged. Dye et al. (2007) have dealt with time proportional backlogging. Everyone of these papers accept a constant interest rate. Circumstances, there are items like unpredictable fluids, drugs, and materials, and so on in which the rate of weakening is expansive. Hence, the misfortune because of disintegration ought not to be disregarded. Aggarwal and Jaggi (1995) broadened Goyal's (2015) model to consider breaking down things. Deficiencies are of incredible significance particularly in a model that considers a postponement in installment because of the way that deficiencies can influence the quantity ordered to profit by the deferral in the chapter.

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Singh et al. (2008) introduced there are 2-weibull distribution inventory demonstrate for decaying things with consistent interest rate where deficiencies were permitted and somewhat multiplied. Singh and Jain (2009) proposed a deterministic inventory show with time shifting disintegration rate and a straight pattern sought after over a finite planning skyline. They accepted that the provider offers a credit cutoff to the retailer amid which no intrigue is charged.

Covert and Philip (1973) 'An EOQ model for items with Weibull distribution deterioration'. Ghare and Schrader's (1963) display and get a financial order quantity show for a variable rate of crumbling by expecting a two-parameter Weibull appropriation, Philip (2014) created EOQ models for things with variable rate of weakening which was additionally summed up by Shah (2015) permitting deficiencies and considering general falling apart capacities. The impact of expansion on inventory management has analyzed by a few creators. Moreover, as discussed in Chakrabarty et al. (1998), An EOQ model for items with Weibull distribution deterioration, shortages and trended demand.

In the paper, we have attempted to build up a two-warehouse inventory framework with a sensible and down to earth disintegration rate. The impact of crumbling of physical merchandise in stock is a sensible component's inventory control. In this model crumbling rate at anything is expected to pursue two-parameter Weibull dispersion capacity of time. This decay rate is appropriate for things with and without life-period. The two warehouse inventory problem is a captivating yet viable subject of choice science.

1.1 Assumptions and Notation

i) The reviving size is consistent and creation is prompt in the midst of supported time span T of each cycle.

ii) Lead time is 0.

iii) Shortage are permitted and completely aggregated.

iv) Demand rate $D(t) = \frac{d}{(e-1)T} e^{\frac{t}{T}}$ at a time t.

v) Deterioration rate $\theta = \alpha\beta t^{\beta-1}$, where $0 < \alpha < 1, \beta \geq 1$

vi) Holding cost $C_1 = h + \gamma t$ per unit.

vii) C_1, C_2 are cost of each item, shortage cost per unit time respectively.

1.2 Mathematical Modeling and Analysis for the System

Let I(t) is the current stock stage at a time t.

$$\frac{dI(t)}{dt} + \theta I(t) = -\frac{d}{(e-1)T} e^{\frac{t}{T}}, \qquad 0 \leq t \leq t_1 \quad \text{……(1.1)}$$

$$\frac{dI(t)}{dt} = -\frac{d}{(e-1)T}e^{\frac{t}{T}}, \quad t_1 \le t \le T \qquad \ldots\ldots\ldots(1.2)$$

Equation (1.1)

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}\cdot I(t) = -\frac{d}{(e-1)}e^{\frac{t}{T}}$$

$$I.F. = e^{\int \alpha\beta t^{\beta-1}dt} = e^{\alpha t^{\beta}}.$$

Solution of equation (1.1) is given by

$$I(t)e^{\alpha t^{\beta}} = -\int \frac{d}{(e-1)T}e^{\frac{t}{T}}e^{\alpha t^{\beta}}dt + B$$

Where "B" the of integration

$$= -\frac{d}{(e-1)T}\int e^{\frac{t}{T}}\left(1+\alpha t^{\beta}\right)dt + B.$$

After expanding $e^{\frac{t}{T}}$ by Taylor's series and neglecting higher order terms of

$\frac{t}{T}$ greater than 1 $\left(\because \frac{t}{T} < 1\right)$, we get

$$I(t)e^{\alpha t^{\beta}} = -\frac{d}{(e-1)T}\int\left(1+\frac{t}{T}\right)\left(1+\alpha t^{\beta}\right)dt + B$$

$$= -\frac{d}{(e-1)T}\int\left(1+\frac{t}{T}+\alpha t^{\beta}+\frac{\alpha}{T}t^{\beta+1}\right)dt + B$$

$$= -\frac{d}{(e-1)T}\left[t+\frac{t^2}{2T}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+2}}{T(\beta+1)}\right] + B. \qquad \ldots\ldots\ldots(1.3)$$

At $t = 0$, $I(t) = S$, then from equation (1.3), we have

$$I(t)e^{\alpha t^{\beta}} = -\frac{d}{(e-1)T}\left[t+\frac{t^2}{2T}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right] + S.$$

… ….. ..(1.4)

At $t = t_1$, $I(t) = 0$, then from equation (84), we have

$$S = \frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}\right].$$

……… .(1.5)

Substituting the value of S in equation (1.4) from equation (1.5), then

$$I(t) = \frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)} - t - \frac{t^2}{2T} - \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{\alpha t^{\beta+2}}{T(\beta+2)}\right](1-\alpha t^{\beta}).$$

… … ... (1.6)

Neglecting higher order terms of α, we get from equation (1.6)

$$I(t) = \frac{d}{(e-1)T}\left[t_1 - t + \frac{t_1^2}{2T} - \frac{t^2}{2T} - \alpha t_1 t^{\beta} + \alpha t^{\beta+1} - \frac{\alpha t_1^2 t^{\beta}}{2T} + \frac{\alpha t^{\beta+2}}{2T} + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha t_1^{\beta+2}}{T(\beta+2)} - \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{\alpha t^{\beta+2}}{T(\beta+2)}\right]$$

$$I(t) = \frac{d}{(\ell-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)} - t - \frac{t^2}{2T} - \alpha t_1 t^{\beta} - \frac{\alpha t_1^2 t^{\beta}}{2T} + \frac{\alpha\beta t^{\beta+1}}{\beta+1} + \frac{\alpha\beta t^{\beta+2}}{2T(\beta+2)}\right]. \quad \ldots \quad \ldots. \ldots\ (1.7)$$

Solution of equation (1.2) is given by

$$I(t) = \frac{d}{e-1}\left(e^{\frac{t_1}{T}} - e^{\frac{t}{T}}\right).$$

………(1.8)

Total amount of deteriorated units

$$D = S - \int_0^{t_1}\frac{d}{(e-1)T}e^{\frac{t}{T}}dt$$

$$= S - \frac{d}{(e-1)T}\left[Te^{\frac{t}{T}}\right]_0^{t_1}$$

$$= S - \frac{d}{(e-1)}\left(e^{\frac{t_1}{T}} - 1\right) \qquad .(1.9)$$

Putting the value of S in equation (1.6), we obtain

$$D = \frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)} - Te^{\frac{t_1}{T}} + T\right].$$

……(1.10)

Number of units in shortage

$$= \int_{t_1}^{T} I(t)dt$$

$$= \frac{d}{e-1}\left[2Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}} - eT\right].$$

Therefore average shortage cost

$$= \frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}} - eT\right] \qquad \ldots.(1.11)$$

Average holding cost

$$= \frac{1}{T}\int_0^{t_1} I(t)(h+\gamma t).dt$$

$$= \frac{1}{T}\int_0^{t_1} hI(t)dt + \frac{1}{T}\int_0^{t_1}\gamma t I(t)dt$$

$$=\frac{h}{T}\frac{d}{(e-1)T}\int_0^{t_1}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t-\frac{t^2}{2T}\right.$$

$$\left.-\alpha t_1 t^{\beta}-\frac{\alpha t_1^2 t^{\beta}}{2T}+\frac{\alpha\beta t^{\beta+1}}{\beta+1}+\frac{\alpha\beta t^{\beta+2}}{2T(\beta+2)}\right]dt$$

$$+\frac{\gamma}{T}\frac{d}{(e-1)T}\int_0^{t_1}\left[t_1 t+\frac{t_1^2 t}{2T}+\frac{\alpha t_1^{\beta+1}.t}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}t-t^2\right.$$

$$\left.-\frac{t^3}{2T}-\alpha t_1 t^{\beta+1}-\frac{\alpha t_1^2 t^{\beta+1}}{2T}+\frac{\alpha\beta t^{\beta+2}}{\beta+1}+\frac{\alpha\beta t^{\beta+3}}{2T(\beta+2)}\right]dt$$

$$=\frac{h}{T}\frac{d}{(e-1)T}\left[t_1^2+\frac{t_1^3}{2T}+\frac{\alpha t_1^{\beta+2}}{\beta+1}+\frac{\alpha t_1^{\beta+3}}{T(\beta+2)}-\frac{t_1^2}{2}-\frac{t_1^3}{6T}\right.$$

$$\left.-\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{\alpha t_1^{\beta+3}}{2T(\beta+1)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)(\beta+3)}\right]$$

$$+\frac{\gamma d}{T(e-1)T}\left[\frac{t_1^3}{2}+\frac{t_1^4}{4T}+\frac{\alpha t_1^{\beta+3}}{2(\beta+1)}+\frac{\alpha t_1^{\beta+4}}{2T(\beta+2)}-\frac{t_1^3}{3}\right.$$

$$\left.-\frac{t_1^4}{8T}-\frac{\alpha t_1^{\beta+3}}{\beta+2}-\frac{\alpha t_1^{\beta+4}}{2T(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$=\frac{h}{T^2}\frac{d}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+1)(\beta+2)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right.$$

$$\left.+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)(\beta+3)}\right]+\frac{y}{T^2}\frac{d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+2)}\right.$$

$$\left.+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$=\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right.$$

$$\left.+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]\right.$$

……..(1.12)

Total average cost per time is

$K(T_1)=\frac{CD}{T}+$ Average Holding cost + Average Shortage cost

$$=\frac{CD}{T}+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+1)(\beta+2)}\right]$$

$$+\frac{C_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right)$$

$$K(T_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$

$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right)$$

……(1.13)

The necessary conditions for minimum the total average costs $K(t_1,T)$ are

$$\frac{\partial K}{\partial T}=0 \qquad \text{And} \qquad \frac{\partial K}{\partial t_1}=0$$

……(1.14)

Now $\frac{\partial K}{\partial T}=0$, gives

$$-2C\left[t_1+\frac{3t_1^2}{4T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{3\alpha t_1^{\beta+2}}{2T(\beta+2)}-\frac{Te^{\frac{t_1}{T}}}{2}-\frac{t_1e^{\frac{t_1}{T}}}{2}+\frac{T}{2}\right]$$

$$-3h\left[\frac{t_1^2}{3}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{2\alpha\beta t_1^{\beta+2}}{3(\beta+1)(\beta+2)}\right]$$

$$-3\gamma\left[\frac{2t_1^3}{9}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$+C_2t_1^2e^{\frac{t_1}{T}}=0.$$

…….(1.15)

And $\frac{\partial K}{\partial t_1}=0$, gives

$$C\left[1+\frac{t_1}{T}+\alpha t_1^{\beta}+\frac{\alpha t_1^{\beta+1}}{T}-e^{\frac{t_1}{T}}\right]$$

$$+h\left[t_1+\frac{t_1^2}{T}+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)}\right]$$

$$+\gamma\left[\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)}\right]$$

$$+C_2\left(Te^{\frac{t_1}{T}}-\frac{t_1}{T}e^{\frac{t_1}{T}}\right)=0.$$

….(1.16)

Equation (1.15) and (1.16) gives the optimum values of T and t_1 respectively. Provided

$$\frac{\partial^2K}{\partial T^2}>0,\quad \frac{\partial^2K}{\partial t_1^2}>0\quad \text{and}\quad \left(\frac{\partial^2K}{\partial T^2}\right)\left(\frac{\partial^2K}{\partial t_1^2}\right)-\left(\frac{\partial^2K}{\partial T\partial t_1}\right)^2>0$$

…

…(1.17)

Case I. in case of finite planning horizon, the complete average cost is

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$

$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$+\frac{C_2d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right.$$

……..(1.18)

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Sub- case 1. When $\beta = 1$ then average cost is

$$K(t_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^2}{2}+\frac{\alpha t_1^3}{3T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^4}{8T}+\frac{\alpha t_1^3}{6}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^4}{24}+\frac{\alpha t_1^5}{30T}\right]+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right)$$

…..(1.19)

$$\frac{dK(t_1)}{dt_1}=\frac{cd}{T^2(e-1)}\left[1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right]$$
$$+\frac{hd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6T}\right]+\frac{c_2 d}{T^2(e-1)}\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)$$

For minimum total average cost $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow\quad C\left(1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0$$

……. (1.20)

Sub- case 2. When $\beta = 2$, deteriration rate become variable linear function of time then total average cost is

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^3}{3}+\frac{\alpha t_1^4}{4T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{2\alpha t_1^5}{15T}+\frac{\alpha t_1^4}{6}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^5}{20}+\frac{\alpha t_1^6}{24T}\right]+\frac{c_2 d}{T^2(e-1)}\left(2T^2e^{\frac{t_1}{T}}-t_1Te^{\frac{t_1}{T}}-eT^2\right)$$

…….(1.21)

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow\quad C\left(1+\frac{t_1}{T}+\alpha t_1^2+\frac{\alpha t_1^3}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{2\alpha t_1^4}{3T}+\frac{2\alpha t_1^3}{3}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{4T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0$$

……..(1.22)

Sub-case 3. When $\beta = 3$, then deterioration rate become quadratic function of time then toatl average cost is

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{5T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^6}{8T}+\frac{3\alpha t_1^5}{20}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^6}{20}+\frac{3\alpha t_1^7}{70T}\right]+\frac{c_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right]$$

…….(1.23)

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow\quad C\left(1+\frac{t_1}{T}+\frac{\alpha t_1^3}{1}+\frac{\alpha t_1^4}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{3\alpha t_1^5}{4T}+\frac{3\alpha t_1^4}{4}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{3\alpha t_1^5}{10}+\frac{3\alpha t_1^6}{10T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0$$

……..(1.24)

For minimum value of $K(t_1)$ $\frac{\partial K}{\partial t_1}=0$, which gives

$$C\left[1+t_1+\alpha t_1^{\beta}+\alpha t_1^{\beta+1}-e^{t_1}\right]+h\left[t_1+t_1^2+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right]$$
$$+\gamma\left[\frac{t_1^2}{2}+\frac{t_1^3}{2}+\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)}\right]$$
$$+C_2\left(Te^{t_1}-t_1e^{t_1}\right)=0$$

…….(1.25)

Case II. When $T = 1$, the total average cost is

$$K(t_1)=\frac{Cd}{e-1}\left[t_1+\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{\beta+2}-e^{t_1}+1\right]$$
$$+\frac{hd}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3}+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$
$$+\frac{\gamma d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)}\right]$$
$$+\frac{C_2 d}{e-1}\left[2e^{t_1}-t_1e^{t_1}-e\right]$$

…….(1.26)

Sub-case 1. When $\beta = 1$, then deterioration rate become constant.

$$K(t_1)=\frac{cd}{e-1}\left[t_1+\frac{t_1^2}{2}+\frac{\alpha t_1^2}{2}+\frac{\alpha t_1^3}{3}-e^{t_1}+1\right]+\frac{hd}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3}+\frac{\alpha t_1^4}{8}+\frac{\alpha t_1^3}{6}\right]+\frac{\gamma d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8}+\frac{\alpha t_1^4}{24}+\frac{\alpha t_1^5}{30}\right]+\frac{C_2 d}{e-1}\left[2e^{t_1}-t_1e^{t_1}-e\right]$$

…..(1.27)

$$\frac{dK(t_1)}{dt_1}=\frac{cd}{e-1}\left[1+t_1+\alpha t_1+\alpha t_1^2-e^{t_1}\right]$$
$$+\frac{hd}{e-1}\left[t_1+t_1^2+\frac{\alpha t_1^3}{2}+\frac{\alpha t_1^2}{2}\right]$$
$$+\frac{\gamma d}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{2}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6}\right]$$
$$+\frac{C_2 d}{e-1}\left[e^{t_1}-t_1e^{t_1}\right]$$

For minimum total average cost $\frac{dK(t_1)}{dt_1}=0$

$\Rightarrow C(1+t_1+\alpha t_1+\alpha t_1^2-e^{t_1})+h\left(t_1+t_1^2+\frac{\alpha t_1^3}{2}+\right.$

αt122+γt122+t132+αt136+αt146+C2et1−t1et1=0

…… (1.28)

<u>Sub - Case 2.</u>

When $\beta = 2$, deteriration rate become variable linear function of time then total average cost is

$$K(t_1)=\frac{cd}{e-1}\left[t_1+\frac{t_1^2}{2}+\frac{\alpha t_1^3}{3}+\frac{\alpha t_1^4}{4}-e^{t_1}+1\right]+\frac{hd}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3}+\frac{2\alpha t_1^5}{15}+\frac{\alpha t_1^4}{6}\right]+\frac{\gamma d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8}+\frac{\alpha t_1^5}{20}+\frac{\alpha t_1^6}{24}\right]+\frac{C_2 d}{e-1}\left[2e^{t_1}-t_1e^{t_1}-e\right]$$

…….(1.29)

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$\Rightarrow C(1+t_1+\alpha t_1^2+\alpha t_1^3-e^{t_1})+h\left(t_1+t_1^2+\frac{2\alpha t_1^4}{3}+\right.$

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$$\left.\frac{\alpha t_1^3}{3}\right) + \gamma\left(\frac{t_1^2}{2} + \frac{t_1^3}{2} + \frac{\alpha t_1^4}{4} + \frac{\alpha t_1^5}{4}\right) + C_2(e^{t_1} - t_1 e^{t_1}) = 0$$

……..(1.30)

Sub-case 3. When β = 3, then deterioration rate become quadratic function of time then toatl average cost is

$$K(t_1) = \frac{cd}{e-1}\left[t_1 + \frac{t_1^2}{2} + \frac{\alpha t_1^4}{4} + \frac{\alpha t_1^5}{5} - e^{t_1} + 1\right] + \frac{hd}{e-1}\left[\frac{t_1^2}{2} + \frac{t_1^3}{3} + \frac{\alpha t_1^6}{8} + \frac{3\alpha t_1^5}{20}\right] + \frac{\gamma d}{e-1}\left[\frac{t_1^3}{6} + \frac{t_1^4}{8} + \frac{\alpha t_1^6}{20} + \frac{3\alpha t_1^7}{70}\right] + \frac{C_2 d}{e-1}\left[2e^{t_1} - t_1 e^{t_1} - e\right] \quad \text{…….(1.31)}$$

$$\frac{dK(t_1)}{dt_1} = \frac{cd}{e-1}(1 + t_1 + \alpha t_1^3 + \alpha t_1^4 - e^{t_1}) + \frac{hd}{e-1}\left(t_1 + t_1^2 + \right.$$

3αt154+3αt144+γde−1t122+t132+3αt1510+3αt161 0+C2de−1et1−t1et1=0

……..(1.32)

For minimum value of $K(t_1)$ $\frac{\partial K}{\partial t_1} = 0$, which gives

$$\Rightarrow C(1 + t_1 + \alpha t_1^3 + \alpha t_1^4 - e^{t_1}) + h\left(t_1 + t_1^2 + \frac{3\alpha t_1^5}{4} + \right.$$

3αt144+γt122+t132+3αt1510+3αt1610+C2et1−t1et 1=0 …….(1.33)

Case III. If γ = 0, then holding cost become constant. The toatl average cost is

$$K(t_1) = \frac{Cd}{T^2(e-1)}\left[t_1 + \frac{t_1^2}{2T} + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha t_1^{\beta+2}}{T(\beta+2)} - Te^{\frac{t_1}{T}} + T\right] + \frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2} + \frac{t_1^3}{3T} + \frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + \frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}} - eT\right) \quad \text{……..(1.34)}$$

For minimum total averages cost, $\frac{dK(t_1)}{dt_1} = 0$

$$\Rightarrow C\left(1 + \frac{t_1}{T} + \alpha t_1^{\beta} + \frac{\alpha t_1^{\beta+1}}{T} - e^{\frac{t_1}{T}}\right) + h\left(t_1 + \frac{t_1^2}{T} + \frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)} + \frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)}\right) + c_2\left(Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}}\right) = 0 \quad \text{………(1.35)}$$

Sub-case 1. When β = 1, deteriration rate become constant.

$$K(t_1) = \frac{Cd}{T^2(e-1)}\left[t_1 + \frac{t_1^2}{2T} + \frac{\alpha t_1^2}{2} + \frac{\alpha t_1^3}{3T} - Te^{\frac{t_1}{T}} + T\right] + \frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2} + \frac{t_1^3}{3T} + \frac{\alpha t_1^4}{8T} + \frac{\alpha t_1^3}{6}\right] + \frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}} - eT\right] \quad \text{………(1.36)}$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1} = 0$

$$\Rightarrow C\left(1 + \frac{t_1}{T} + \alpha t_1 + \frac{\alpha t_1^2}{T} - e^{\frac{t_1}{T}}\right) + h\left(t_1 + \frac{t_1^2}{T} + \frac{\alpha t_1^3}{2T} + \frac{\alpha t_1^2}{2}\right) + C_2\left(Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}}\right) = 0 \quad \text{……..(1.37)}$$

Sub- case 2. When β = 2, deteriration rate become variable linear function of time.

$$K(t_1) = \frac{Cd}{T^2(e-1)}\left[t_1 + \frac{t_1^2}{2T} + \frac{\alpha t_1^3}{3} + \frac{\alpha t_1^4}{4T} - Te^{\frac{t_1}{T}} + T\right] + \frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2} + \frac{t_1^3}{3T} + \frac{2\alpha t_1^5}{15T} + \frac{\alpha t_1^4}{6}\right] + \frac{C_2 d}{T(e-1)}\left[2T^2 e^{\frac{t_1}{T}} - t_1 Te^{\frac{t_1}{T}} - eT^2\right]$$

……..(1.38)

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1} = 0$

$$\Rightarrow C\left(1 + \frac{t_1}{T} + \alpha t_1^2 + \frac{\alpha t_1^3}{T} - e^{\frac{t_1}{T}}\right) + h\left(t_1 + \frac{t_1^2}{T} + \frac{2\alpha t_1^4}{3T} + \frac{2\alpha t_1^3}{3}\right) + C_2\left(Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}}\right) = 0 \quad \text{……..(1.39)}$$

Sub-case 3. When β = 2, deteriration rate become quadratic function of time then total average cost is

$$K(t_1) = \frac{Cd}{T^2(e-1)}\left[t_1 + \frac{t_1^2}{2T} + \frac{\alpha t_1^4}{4} + \frac{\alpha t_1^5}{5T} - Te^{\frac{t_1}{T}} + T\right] + \frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2} + \frac{t_1^3}{3T} + \frac{\alpha t_1^6}{8T} + \frac{3\alpha t_1^5}{20}\right] + \frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}} - t_1 Te^{\frac{t_1}{T}} - eT\right] \quad \text{……… (1.40)}$$

For minimum total average cost, $\frac{dK(t_1)}{dt_1} = 0$,

$$\Rightarrow C\left(1 + \frac{t_1}{T} + \alpha t_1^3 + \frac{\alpha t_1^4}{T} - e^{\frac{t_1}{T}}\right) + h\left(t_1 + \frac{t_1^2}{T} + \frac{3\alpha t_1^5}{4T} + \frac{3\alpha t_1^4}{4}\right) + C_2\left(Te^{\frac{t_1}{T}} - t_1 e^{\frac{t_1}{T}}\right) = 0 \quad \text{……..(1.41)}$$

II. CONCLUSION

In the paper, we have attempted to evolve a two-warehouse inventory structure too an exceptionally reasonable practical disintegrating rate. The impact of debilitating of physical items in stock is extraordinarily reasonable component's stock control. Now in the model decay rate at anything is relied upon to seek after two parameter Weibull scattering limit of time. This debilitating rate is appropriate for things with and without life-period. The two warehouse stock issue is a beguiling yet feasible purpose of decision science. The two-warehouse model can be associated with various sensible conditions, in light of introduction of open market technique; the business contention ends up being very high to have most prominent possible market.

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