

# Relaxed Skolem Mean Labeling of Four Star Graphs $K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$ Where $l_1 \leq l_2 \leq l_3$ With $|l - l_1 - l_2 - l_3| = 1$

Abraham K Samuel, J. Vinolin, D. S. T. Ramesh

Abstract: The four star graphs  $G = K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l_k} M_{1,l_k}$ where  $l_1 \leq l_2 \leq l_3 \ l_3$  will be relaxed skolem mean if  $|l - l_1 - l_2 - l_3| = 1$  I is the main purpose of this article.

Keywords: Relaxed Skolem mean graph, relaxed skolem mean labeling, star graphs.

### I. INRODUCTION

Relaxed skolem mean labeling was first conceived by V.Balaji et. al.[3] in the year 2010. In that paper [3] he introduced the definition of relaxed skolem mean labeling for the first time and also some basic properties for a graph to be relaxed skolem mean. The most important properties are (i) If G is a graph with n vertices and m edges then G is said to be a relaxed skolem mean graph only if  $n \ge m$ , (ii)  $G = K_{1,n}$  where  $n \ge 5$  is not a relaxed skolem mean graph.

### **II. PRELIMINARIES**

**Definition 1:** A graph label is the assigning of labels to edges and also vertices of a graph or simply either edges or vertices of a graph only by integers.

**Notation:** If x is a real number, the integral part of x is denoted by [x] which is the largest integer less than or equal to x.

We define the relaxed skolem mean labeling of a graph G of order p and size q as follows:

Revised Manuscript Received on February 05, 2020.

\* Correspondence Author

**Abraham K Samuel\***, Research Scholar, Department of Mathematics, St. Xavier's College, Palayamkottai, Tirunelveli-627002, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India. abksamkottayam@gmail.com.

**J.Vinolin**, Research Scholar, Department of Mathematics Reg. No. 12310, St. Xavier's College, Palayamkottai, Tirunelveli-627002, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India.judevino3010@gmail.com.

**D.S.T.Ramesh**, Department of Mathematics, Nazareth Margoschis College, Pillaiyanmanai,Thoothukudi-628617, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, TamilNadu, India. dstramesh@gmail.com

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an <u>open access</u> article under the CC BY-NC-ND license (<u>http://creativecommons.org/licenses/by-nc-nd/4.0/</u>)

**Definition 2:** The vertex labeling  $f : V \rightarrow \{1, 2, ..., p+1\}$  and the induced edge labeling  $f^*: E \rightarrow \{2,3..., p+1\}$  is a relaxed skolem mean labeling if both *f* and  $f^*$  are one – one functions and  $f^*(e = uv) = [(f(u)+f(v)+1)/2].$ 

**Note:** Graph G has p vertices and the available vertex labels are p+1. Therefore, one number from the set {1, 2, 3, ..., p + 1} will not be used to label any vertex of G. We call that number as the relaxed label. If the relaxed label is p + 1, the relaxed mean labeling becomes the Skolem mean labeling.

#### III. MAIN RESULT

**Theorem 1:** Four star graph  $G = K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$ where  $l_1, l_2, l_3$  are in ascending order is a relaxed skolem mean graph if  $||-l_1-l_2-l_3| = 1$ .

**Proof**: Let  $L_k = \sum_{j=1}^k l_j$ ;  $1 \le k \le 3$ . Hence we have  $L_1 = l_1$ ;  $L_2 = l_1 + l_2$  and  $L_3 = l_1 + l_2 + l_3$ .

Consider the graph  $G = K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l_1}$ .

Let  $V = V_1 \cup V_2 \cup V_3 \cup V_4$  be the set of vertices of G where  $V_k = \{v_{k,i}: 0 \le i \le l_k\}$  for  $1 \le k \le 3$ ,  $V_4 = \{v_{4,i}: 0 \le i \le l\}$ . Let  $E = \bigcup_{k=1}^3 \{v_{k,0}v_{k,i}: 1 \le i \le l_k\} \cup \{v_{4,0}v_{4,i}: 1 \le i \le l\}$ be the set of edges of G. The condition  $|l \cdot l_1 \cdot l_2 \cdot l_3| = 1 \Longrightarrow$  $|l - \sum_{j=1}^3 l_j| = 1 \Longrightarrow |l - L_3| = 1$ That is, there are two cases

viz.  $l = L_3 + 1$  and  $l = L_3 - 1$ .

**Case I:**  $l = L_3 + 1$ .

G has  $L_3 + l + 4 = 2L_3 + 5$  vertices and  $L_3 + l = 2L_3 + 1$  edges. The vertex labeling

 $f: V \to \{1, 2, 3, \dots, p + 1 = L_3 + l + 4 + 1 = 2L_3 + 6\}$  given as:

 $f(v_{1,0}) = 1; f(v_{2,0}) = 2; f(v_{3,0}) = 4$   $f(v_{4,0}) = L_3 + l + 5 = 2L_3 + 6$   $f(v_{1,k}) = 2k + 4 \qquad 1 \le k \le l_1$   $f(v_{2,k}) = 2L_1 + 2k + 4 \qquad 1 \le k \le l_2$   $f(v_{3,k}) = 2L_2 + 2k + 4 \qquad 1 \le k \le l_3$   $f(v_{4,k}) = 2k + 1 \qquad 1 \le k \le l = L_3 + 1$ Here  $2L_3 + 5$  is the relaxed label.

Their induced labels for edges are as follows:

The induced label of  $v_{1,0}v_{1,k}$  is 3 + k where  $1 \le k \le l_1$ (edge labels are 4,5, ....,  $l_1 + 3 = L_1 + 3$ ),  $v_{2,0}v_{2,k}$  is  $L_1 + 3 + k$  for  $1 \le k \le l_2$  (edge labels

are  $L_1 + 4$ ,  $L_1 + 5$ , ...,  $L_1 + l_2 + 3 =$ 

& Sciences Publication

Blue Eyes Intelligence Engineering

Published By:



Retrieval Number: C6354029320/2020©BEIESP DOI: 10.35940/ijeat.C6354.029320 Journal Website: <u>www.ijeat.org</u>

# Relaxed Skolem Mean Labeling Of Four Star Graphs $K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$ Where $l_1 \le l_2 \le l_3$ With $|l - l_1 - l_2 - l_3| = 1$

 $L_2 + 3$ ),  $v_{3,0}v_{3,k}$  is  $L_2 + 4 + k$  for  $1 \le k \le l_3$ 

(edge labels are  $L_2 + 5$ ,  $L_2 + 6$ , ...,  $L_2 + l_3 + 4 = L_3 + 4$ ),  $v_{4,0}v_{4,k}$  is  $L_3 + 4 + k$  for  $1 \le k \le l$  (edge labels are  $L_3 + 5$ ,  $L_3 + 6$ , ...,  $L_3 + l + 5 = L_3 + (L_3 + 1) + 4 = 2L_3 + 5$ ).

The labels of edges induced by the labels of vertices of graph G are distinct. This shows that G is a relaxed skolem mean . Case II:  $l = L_3 - 1$ 

G has  $L_3 + l + 4 = 2L_3 + 3$  vertices and  $L_3 + l = 2L_3 - 1$  edges. The vertex labeling  $f: V \rightarrow \{1, 2, 3, ..., p + 1 = L_3 + l + 4 + 1 = 2L_3 + 5\}$ given as:  $f(v_{1,0}) = 1$ ;  $f(v_{2,0}) = 2$ ;  $f(v_{3,0}) = 3f(v_{4,0}) = L_3 + l + 5 = 2L_3 + 4$   $f(v_{1,k}) = 2k + 2$   $f(v_{2,k}) = 2L_1 + 2k + 2$   $f(v_{3,k}) = 2L_2 + 2k + 2$   $f(v_{3,k}) = 2L_2 + 2k + 2$   $f(v_{4,k}) = 2k + 3$   $1 \le k \le l_3$   $f(v_{4,k}) = 2k + 3$  $1 \le k \le l = L_3 - 1$ 

Here  $2L_3 + 3$  is the relaxed label.

Their induced labels for edges are as follows:

The induced label of  $v_{1,0}v_{1,k}$  is 2 + k where  $1 \le k \le l_1$ (edge labels are 3,4,...,  $l_1+2 = L_1+2$ ),  $v_{2,0}v_{2,k}$  is  $L_1+2+k$ for  $1 \le k \le l_2$  (edge labels are  $L_1+3$ ,  $L_1+4$ , ...,  $L_1+l_2+2 = L_2+2$ ),  $v_{3,0}v_{3,k}$  is  $L_2+3+k$  for  $1 \le k \le l_3$  (edge labels are  $L_2+4$ ,  $L_2+5$ , ...,  $L_2+l_3+3 = L_3+3$ ),  $v_{4,0}v_{4,k}$  is  $L_3+4+k$  for  $1 \le k \le l$  (edge labels are  $L_3+5$ ,  $L_3+6$ , ...,  $L_3+l+5 = L_3+(L_3-1)+4 = 2L_3+3$ ).

The labels of edges induced by the labels of vertices of graph G are distinct.

This shows that G is a relaxed skolem mean . is skolem mean We illustrate the above two cases with the following four star graphs.

Fig.1. is an illustration of Case I where  $l = L_3 + 1$ 



Fig. 2. is an illustration of Case II where  $l = L_3 - 1$ 



# **IV. CONCLUSION**

Relaxed skolem mean labeling of a four star graph  $G = K_{1,l_1} \bigcup K_{1,l_2} \bigcup K_{1,l_3} \bigcup K_{1,l}$  is discussed in this paper . We mainly discussed the two cases  $l = l_3 + 1$  and  $l = l_3 - 1$  and gave the labeling which satisfies the condition of relaxed skolem mean labeling which exists for graph G. We are in further research to find upto how many cases the graph G will allow relaxed skolem mean labeling.

# REFERENCE

- M. Apostal, "Introduction to Analytic Number Theory", Narosa Publishing House, Second edition, 1991.
- J. A. Bondy and U. S. R. Murty, "Graph Theory with Applications", Macmillan press, London, 1976.
- J. C. Bermond," Graceful Graphs, Radio Antennae and French Wind Mills", Graph Theory and Combinatories, Pitman, London, 1979, 13 – 37.
- V. Balaji, D. S. T. Ramesh and A. Subramanian, "Skolem Mean Labeling", Bulletin of Pure and Applied Sciences, vol. 26E No. 2, 2007, 245 – 248.
- V. Balaji, D. S. T. Ramesh and A. Subramanian, "Some Results On Skolem Mean Graphs", Bulletin of Pure and Applied Sciences, vol. 27E No. 1, 2008, 67 – 74.
- V. Balaji, D. S. T. Ramesh and A. Subramanian, "Relaxed Skolem Mean Labeling", Advances and Applications in Discrete Mathematics, vol. 5(1), January 2010, 11 – 22.
- V. Balaji, D. S. T. Ramesh and A. Subramanian, "Some Results On Relaxed Skolem Mean Graphs", Bulletin of Kerala Mathematics Association, vol. 5(2), December 2009, 33 – 44.
- J. A. Gallian, "A Dynamic Survey of Graph Labeling", The Electronic Journal of combinatorics 14(2007).

# **AUTHORS PROFILE**



Abraham K Samuel, M.Sc., M.Phil., Research Scholar, St. Xavier's College, Palayamkottai, Tamil Nadu. He is an Associate Professor also the head of the department of MathematicsCollege, Kottayam, Kerala Affiliated to M. G. University Kerala. He has 27 years of teaching experience.



Published By:

& Sciences Publication

Blue Eyes Intelligence Engineering

**J.Vinolin, M.Sc., M.Phil.,** Research Scholar, St. Xavier's College, Palayamkottai, Tamil Nadu. Published two papers in National and International Journals. Presented 3 papers in three National Conferences.



Retrieval Number: C6354029320/2020©BEIESP DOI: 10.35940/ijeat.C6354.029320 Journal Website: <u>www.ijeat.org</u>





Dr. D.S.T.Ramesh, M.Sc., M.Phil., Ph.D., He is an Associate Professor of Mathematics in Nazareth Margoschis College, Pillayanmanai, Tuticorin, Tamil Nadu. He has 33 years of teaching experience. He guided 8 Ph.D."s and 5 M. Phil. His area of specialization is Graph Theory. He published 51 research papers in National and International Journals. He was invited as a

resource person for National conferences organized by Sacred Heart College, Tirupattur and Pope's College, Sawyerpuram.



Retrieval Number: C6354029320/2020©BEIESP DOI: 10.35940/ijeat.C6354.029320 Journal Website: <u>www.ijeat.org</u>