

Selection and Analysis of Input-Output Variables using Data Envelopment Analysis of Decision Making Units - Indian Private Sector Banks

B. Vittal, Raju Nellutla, M. Krishna Reddy



Abstract: In banking system the evaluation of productivity and performance is the key factor among the fundamental concepts in management. For identify the potential performance of a bank efficiency is the parameter to evaluate effective banking system. To measure the efficiency of a bank selection of appropriate input-output variables is one of the most vital issues. The suitable identification of input-output variables helps to create and identify model in order to evaluate the efficiency and analysis. The Data Envelopment Analysis (DEA) is a mathematical approach used to measure the efficiency of identified Decision Making Units (DMUs). The DEA is a methodology for evaluating the relative efficiency of peer decision making units of identified input/output variables for the financial year 2018-19. In this study the basic DEA CCR, BCC models used for measure the efficiency of DMUs. In addition to these models for minimize the input excess and output shortfall Slack Based Measure (SBM) efficiency used. The SBM is a scalar measure which directly deals with slacks of input, output variables which help in obtain improved efficiency score compare with previous model. The result from the analysis is

Keywords: DEA, Efficiency, Input variables, Output variables, CCR, BCC and SBM

I. INTRODUCTION

The Data Envelopment Analysis is non-parametric linear programming problem used to measure the relative efficiency of Decision Making Units (DMUs) of identified multiple input, output variables[1] of Private Sector Banks (Pvt.SBs).

The output performance metrics of DMUs are classified as DEA inputs and outputs (Charnes et al. 1978). DEA exhibits a set of DMUs into a set of efficient DMUs which form a best practice to the frontier line of production possibility set and a set of inefficient DMUs[3]. In the DEA if the performance of inefficient DMUs diminishes or improves, the efficient DMUs still may get its efficiency unity score. However the performance of inefficient DMUs always depends on the efficient DMUs, efficient DMUs are only characterized by a unity efficiency score.

The DEA is the benchmark procedure used to measure efficient DMUs from the set of given DMUs. In this sense, all DMUs under this approach are being benchmarked against the identified DEA on the frontier line of the production possibility set.

As the best practice of benchmarking, efficient DMUs are benchmark to the inefficient DMUs as these DMUs having its efficiency score is less than 1.

Hence, the efficiency score of identified DMUs lies between 0 and 1.

In the Data Envelopment Analysis for measure the efficiency of DMUs the basic model is CCR (Charnes, Cooper, Rhode) given in the year 1978. This approach used to measure the efficiency from the ratio of multiple outputs to input of the DMUs for the analysis. The main objective of the CCR model is the minimum input which satisfies at least the given output level and maximizes the output without considerable level of observed input values. The basic assumption of CCR is Constant Returns-to-Scale (CRS) i.e., the proportional change in the input (output) and it proceeded and followed by the same direction of proportional change in its output (input) of the DMUs.

Improvement in obtaining the efficiency score by Banker from the CCR model is BCC (Banker, Charnes, and Cooper) modelled efficiency approach in the year 1984[2]. The BCC approach used to measure the efficiency of DMUs from virtual outputs to the virtual inputs of given DMUs of input, output variables. The basic assumption of BCC model is Variable Returns-to-Scale (VRS) is represents the Decision Making Units of variables based nature it may increase, diminishes or constant on the frontier line of production possibility set. The comparative study of CCR, BCC Model is made on frontier line, it exhibit BCC approach having better efficiency scores than CCR due to VRS assumption. The subsequent improvement in calculating efficiency scores by minimizing the slacks of input, output variables given by Kavoru Tone (1997, 2001) is Slack Based Measure (SBM). According to this approach the CCR, BCC approach fails to attain input excess and output shortfall which gives the result as non-zero slack and it is proceeded to follow the evaluation of radial (proportional) efficiency of the $DMU\theta^*$ [5]. The slack based measure of efficiency evaluates the efficiency of DMUs by minimizing its slack and it is invariant to unit measure. SBM directly effects on the Non-zero slacks and it tries to minimize the non-zero slack of the DMUs. The main objective of the SBM is by minimizing the non-zero slack and improving the efficiency of DMUs slack but it does not affected the whole data set. The SBM model is designed to estimate[6]

Manuscript received on May 21, 2021.

Revised Manuscript received on May 27, 2021.

Manuscript published on June 30, 2021.

* Corresponding Author

B. Vittal*, Assistant Professor, Department of Computer Science and Engineering, CVR College of Engineering, Hyderabad(Telangana), India.

Raju Nellutla, Associate Professor, Department of Humanities and Science, Gurunanak Institutions, Hyderabad(Telangana), India.

M. Krishna Reddy, Professor, Department of Computer Science and Engineering, CVR College of Engineering, Hyderabad(Telangana), India.

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- (1) In-variant with respect to the units of the data.
- (2) Monotone decreasing in the slacks of input and output.

II. THEORY AND METHODOLOGY

In the present study there are 21 Pvt.SBs in India are taken as decision making units. The output from the two inputs, six output variables obtained using Sai Tech Inc. DEA Solver software used. Identification of input, output variables in DEA analysis is the challenging task due to misleading in nature. For this selection and analysis from the variables, the data has been taken for the period 2018-19.

The analysis of the following set of variables is quantifying the efficiency of DMUs in India.

Input variables: Borrowing, Number of Employees

Output variables: Deposits, Investment, Loan & Advances, NPAs, Net Profits and Net Income.

For selection/eliminate the input, output variables Step wise method used for identify appropriate input, output variables using CCR Model.

A. CCR Model

The productivity and Technical efficiency terms used in the DEA such that the production acts as transforming its input into outputs because the objective of the production is to create the values through transforming from the input to desirable outcomes (outputs)[7]. The production technology creates a function using input and output variables.

The CCR (1981) introduced a method of the DEA to deal with the problem of efficiency measurement for DMUs with multiple input and output variables[4]. Suppose, there are N firms from the production possibility set, each yield the m outputs from n inputs. Firm s uses the input function $x^s = (x_{1s} x_{2s} \dots x_{ns})$ to produce the output function $y^s = (y_{1s} y_{2s} \dots y_{ms})$. The average productivity measure of productivity of the given firms as follows

$$AP_s = \frac{\sum_{r=1}^m u_{rs} y_{rs}}{\sum_{i=1}^n v_{is} x_{is}}$$

In the DEA from the production possibility set, no average productivity of the firm s more than unity. From this case, the productivity function formulates as follows

$$P_s = \frac{\sum_{r=1}^m u_{rs} y_{rs}}{\sum_{i=1}^n v_{is} x_{is}} \leq 1 \quad (j = 1, 2, \dots, N)$$

$$u_{is} \geq 0; (i = 1, 2, \dots, n); \quad v_{rs} \geq 0; (r = 1, 2, \dots, m)$$

Applying Charnes, Cooper transformation (1984) to the above fractional programming problem can be transformed into a linear programming problem at input minimization function is as follows

$$\lambda(\text{CCR}) = \text{Min } \lambda$$

$$\text{Subject to } \sum_{r=1}^s \lambda_j x_{ij} \leq \lambda x_{ij}$$

$$\sum_{i=1}^m \lambda_j y_{rj} \geq y_{rj} \quad j = 1, 2, 3, \dots, n.$$

$$\lambda_j \geq 0$$

$$y_{rj} \rightarrow S^{\text{th}} \text{ Output for } n^{\text{th}} \text{ DMU}$$

$$x_{ij} \rightarrow m^{\text{th}} \text{ input for } n^{\text{th}} \text{ DMU}$$

From the fundamental theorem of duality the objective functions are equal

$$\text{Max } \sum_{r=1}^s u_r y_{rj_0} = \text{Min } \lambda$$

The objective of the CCR is it minimizes the input which satisfying at least the given output level and maximize the output without considerable level of observed input values. CCR approach follows the Constant Returns Scale (CRS) i.e., the proportional change in the input and it followed by the proportionate change in its output.

Definition 1: The optimum solution of linear problem satisfies to call it is a CCR-efficient

$$(i) \quad \theta^* = 1$$

$$(ii) \quad \text{All slacks are zero} \quad (s^{-*} = 0, \quad s^{+*} = 0) \\ \text{Otherwise CCR-inefficient. CCR approach fails to attain Variable returns scale.}$$

B. Drawback of CCR Model

The drawback of radial-CCR model is neglect the non-radial slack while projecting the efficiency score θ^* . In general we have in many cases non-radial slack play a vital role in exhibiting the efficiency. In this case this model may mislead the possible decision while utilize the efficiency score θ^* as the only measure for evaluating performance of DMUS.

C. BCC Model

The existing approach CCR extended by Banker, charnes, Chooper (BCC) in the year 1984. This approach follows variable returns scale (VRS), the objective of this model is of increasing, decreasing and constant characteristics at different point on the production frontier. The production frontier of BCC model exposed the convexity condition

$$\sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \geq 0 \text{ in its constraints.}$$

The minimal input oriented BCC is given by

$$\theta^*(\text{BCC}) = \min(\theta)$$

$$\text{Subject to } \theta^* x_0 - X\lambda \geq 0$$

$$Y\lambda \geq y_0$$

$$e\lambda = 1$$

$$\lambda \geq 0$$

Where, θ^* is a scalar.

The BCC is said to be efficient if it satisfies the following definition holds good

Definition: If an optimal solution obtained in this process for (BCC_0) satisfies it has no slack ($s^{-*} = 0, \quad s^{+*} = 0$) and $\theta^* = 1$, then DMU_0 is said to be efficient.

D. Drawback of BCC Model

The above linear programming may suffer from slacks of the identified variables comparatively. Due to the slacks, the results obtained from this approach may not be reliable of the Pvt.

SBs and BCC approach not much involve in reducing the account of input excesses and output shortfalls that leads to the non-zero slacks. To eliminate such non-zero slack deficiency, Tone introduced (1997, 2001) a model is called Slack-Based Measure (SBM) of efficiency. This methodology considers the account of the input excesses and output shortfalls that leads to the non-zero slacks. It indicates the drawback because of θ^* does not include the non-zero slacks (2002).

E. Slack Based Measure (SBM) of Efficiency

Let us consider with n DMUs corresponding input and output indices $X = (x_{ij})$ and $Y = y_{ij}$ respectively. Here we assumed that $(X, Y) > 0$

The production possibility set P is given by

$$P = \{(x, y) / x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}$$

Where λ is a nonnegative vector

Consider the expression of a certain DMU (x_0, y_0) as

$$x_0 = X\lambda + s^-$$

$$y_0 = Y\lambda - s^+$$

With $\lambda \geq 0, s^- \geq 0$ and $s^+ \geq 0$. The slacks s^-, s^+ represents the input surplus and output shortage, respectively. Using slack and surplus behaviour the index ρ is given as follows

$$\rho = \frac{1 - \frac{1}{k} \sum_{i=1}^k s_i^- / x_{i0}}{1 + \frac{1}{m} \sum_{i=1}^m s_i^+ / y_{i0}}$$

The above function should satisfies the property of Unit invariant and Monotone [12] and the function should satisfies the range of SBM ρ

$$0 < \rho < 1.$$

Hence the formulation of linear fractional program in λ, s^-, s^+

$$\text{Min } \rho = \frac{1 - \frac{1}{k} \sum_{i=1}^k s_i^- / x_{i0}}{1 + \frac{1}{m} \sum_{i=1}^m s_i^+ / y_{i0}}$$

$$\text{Subject to } x_0 = X\lambda + s^-$$

$$y_0 = Y\lambda - s^+$$

$$\text{With } \lambda \geq 0, s^- \geq 0 \text{ and } s^+ \geq 0.$$

The above linear fractional programming can be used with the combination of constant and variable returns scale. Slack based measure hold good if it satisfies the following

Definition: A DMU (x_0, y_0) is SBM-efficient in its $\rho^* = 1$ and $s^{*-} = 0$ and $s^{*+} = 0$ i.e., there is no input excess and output shortfall [12] in the optimum solution (2009). For an SBM-inefficient, the DMU (x_0, y_0) can be expressed as follows

$$x_0 = X\lambda + s^-$$

$$y_0 = Y\lambda - s^+$$

From the above expansion, the DMU (x_0, y_0) can expect improvement and this leads to become efficient DMU by deleting the input excesses and output shortfalls.

F. SBM CCR Model

Slack-based measure under CCR Model can be formulated as follows

$$(\text{CCR}) \text{ Min } \theta$$

$$\text{Subject to } \theta x_0 = X_\mu + t^-$$

$$y_0 = Y_\mu - t^+$$

$$\mu \geq 0, t^{-1} \geq 0, t^+ \geq 0.$$

The optimum solution of (CCR) is $(\theta^*, \mu^*, t^{*-}, t^{*+})$ obtained by

$$x_0 = X\mu^* + t^{*-} + (1 - \theta^*)x_0$$

$$y_0 = Y\mu^* - t^{*+}.$$

Thus, (λ, s^-, s^+) is feasible for (SBM) and the objective value can be expressed as follows

$$\rho = \frac{\theta^* - \frac{1}{k} \sum_{i=1}^k t_i^{*-} / x_{i0}}{1 + \frac{1}{m} \sum_{i=1}^m t_i^{*+} / y_{i0}}$$

Theorem: Tone (1997) A DMU (x_0, y_0) is CCR-efficiency if and only if it is SBM-efficient (2010) [10].

Definition-1: $\theta = 1$ and $(t^- = s^-, t^+ = s^+) \neq (0, 0)$. In this case, an optimum solution for (CCR) is inefficient.

Definition-1: $\theta = 1$ and $(t^- = s^-, t^+ = s^+) = (0, 0)$. In this case, an optimum solution for (CCR) is efficient.

Definition-3: $\theta < 1$. In this case, (x_0, y_0) is CCR-inefficient.

G. SBM BCC Model

Slack-based measure under BCC Model can be formulated as follows

$$(\text{BCC}) \text{ Min } \theta$$

$$\text{Subject to } \theta x_0 = X_\mu + \lambda + t^-$$

$$y_0 = Y_\mu + \lambda - t^+$$

$$\mu \geq 0, t^{-1} \geq 0, t^+ \geq 0.$$

The optimum solution of (BCC) is $(\theta^*, \mu^*, t^-, t^+)$ obtained by

$$x_0 = X\mu^* + t^- + (1 - \theta^*)x_0$$

$$y_0 = Y\mu^* - t^+.$$

Thus, (λ, s^-, s^+) is feasible for (SBM) and the objective value can be expressed as follows

$$\rho = \frac{\theta^* - \frac{1}{k} \sum_{i=1}^k t_i^- / x_{i0}}{1 + \frac{1}{m} \sum_{i=1}^m t_i^+ / y_{i0}}$$

Theorem: Tone (1997) A DMU (x_0, y_0) is BCC-efficiency if and only if it is SBM-efficient (2010)[9].

Definition-1: $\theta = 1$ and $(t^- = s^-, t^+ = s^+) \neq (0, 0)$. In this case, an optimum solution for (BCC) is inefficient.

Definition-1: $\theta = 1$ and $(t^- = s^-, t^+ = s^+) = (0, 0)$. In this case, an optimum solution for (BCC) is efficient.

Definition-3: $\theta < 1$. In this case, (x_0, y_0) is BCC-inefficient.

III. RESULTS AND DISCUSSIONS

A. Stepwise – Method

The stepwise approach is to select the outputs from the possible number of outputs is started with two inputs and six outputs[8]. The BCC (1984) model formulated and solved individually for 21 Indian Pvt.SBs using stepwise procedure to identify suitable input, output variables for measure efficiency and analysis in DEA.

***	Mean efficiency	Efficient DMUs	Average Efficiency Change
Start (2I, 6 O)	0.9513	15	--
Variables Dropped			
Loan & Advances	0.9394	14	0.0119
NPA's	0.9397	13	0.0116
Deposits	0.9384	14	0.0129

Investment	0.9320	15	0.0193
Net Profit	0.9512	15	0.0001
Net Income	0.9491	15	0.0022

[I – Represents Input variables, O-Represents Output Variables]

The least influenced output variable on efficient DMUs is **Net Profit, Net Income**. The variable Net Profit and Net Income are least influenced variable on efficient DMUs with the average efficiency change 0.0001 and 0.0022 which can be dropped from the DEA efficiency exploration.

	Mean efficiency	Efficient DMUs	Average Efficiency Change
Start (2I, 4 O)	0.9398	14	--
Variables Dropped			
Loan & Advances	0.9231	13	0.0167
Deposits	0.9182	13	0.0216
Investment	0.8960	13	0.0438
NPA's	0.9283	12	0.0115

From the above results, the output variables shown least impact on the efficient DMU is NPA's. The variable NPA's is dropped from the DEA exploration, which has lesser impact on efficient DMUs comparing to the other variables with the mean efficiency change 0.0115.

From the above result of step wise procedure, three output variables are dropped by fixing two input variables. Using two inputs, three output variables efficiency analysis can be made. Finally from the stepwise analysis we choose the following input and output variables for further DEA exploration are as follows

Inputs:

1. Number of Employees
2. Borrowing

Outputs:

1. Loan & Advances
2. Deposits
3. Investment

B. Output of Slack Based Measure of Efficiency

Table - I: Efficiency Benchmark of SBM CRS method of DMUs using DEA

S. No	DMU	Score	Rank	CRS Benchmark (Lambda)	CRS Peer
1	Axis	0.5534	17	Federal (2.926); Kotak Mahindra (0.03); Tamilnad Mercantile (4.177)	
2	Catholic Syrian Bank Ltd.	1	1	Catholi Syrian	1
3	City	1	1	City	3
4	DCB	0.4549	20	J And K Bank (0.295); Kotak Mahindra (0.001); Tamilnad Mercantile (0.048)	
5	Dhanalaxmi	1	1	Dhanalaxmi	0

6	Federal	1	1	Federal	5
7	HDFC	0.7233	14	Federal Bank (5.029); Kotak Mahindra (0.043); Tamilnad Mercantile (0.681)	
8	ICICI	0.4997	18	Federal (3.499); Kotak Mahindra (0.034); Tamilnad Mercantile (4.923)	
9	IndusInd	0.4663	19	Federal (1.123); TamilnadMercantile(1.233)	0
10	IDBI	1	1	IDBI	3
11	J & K Bank	1	1	J & K Bank	4
12	Karnataka	0.7867	13	City (0.22); J & K Bank (0.665); Kotak Mahindra (0.002)	
13	Karur Vysya	0.9969	10	City (0.443); J & K Bank (0.49); Kotak Mahindra (0.002)	
14	Kotak Mahindra	1	1	Kotak Mahindra	7
15	Lakshmi Vilas	0.9387	11	City (0.086); J & K Bank (0.335)	
16	Nainital	0.3098	21	IDBI (0.025)	
17	South Indian	0.8318	12	Federal (0.232); J & K Bank (0.547)	
18	Thamilnad Mercantile	1	1	Thamilnad Mercantile	7
19	Yes	0.6271	16	IDBI (0.419); Kotak Mahindra (0.053); Tamilnad Mercantile (0.639)	
20	Bandhan	1	1	Bandhan	0
21	IDFC	0.6764	15	IDBI (0.468); J & K Bank (0.998)	

The private sector banks are exposed to a common production frontier. The objective of SBM CCR is Constant Returns-to-Scale (CRS) and it assumed to be constant. The largest θ (SBM) efficiency score 1 imply DMU(s) is technically efficient and the rest of the DMUs are inefficient whose efficiency score is less than 1.

From the output of this approach, the efficient banks are Catholic Syrian (DMU 2), City (DMU 3), Dhanalakshmi (DMU 5), Federal (DMU 6), IDBI (DMU 10), J & K (DMU 11), Kotak Mahindra (DMU 14), Thamil Mercantile (DMU 18) and Bandhan (DMU 20) Bank. These banks are efficient under Constant returns to scale on frontier line of production possibility set and these are the benchmark to the inefficient banks. We see that under SBM CRS results out of 21 DMUs, 9 DMUs are efficient and 12 are the inefficient DMUs[11].

The result from the SBM CRS approach, the banks whose efficiency is decline under constant returns to scale is Nainital (DMU 16) bank whose efficiency score is 0.3098. This DMU technically 69% need to improve to become an efficient DMU. Similarly, the other inefficient DMUs under SBM CRS approach are DCB (DMU 4) (0.4549), IndusInd (DMU 9) (0.4663), ICICI (DMU 8) (0.4997) and Axis (DMU 1) (0.5534) bank. A remarkable thing from the efficiency score is that the largest commercial bank ICICI, HDFC showed poor performance as its efficiency score is decline at SBM CRS. To become an efficient DMU, the ICICI need to recover 41.03% of its efficiency score

without increasing its input and in the similar way HDFC Bank need 27.67% of improvement in its efficiency score to become an efficient DMU. Under this model, Karur Vysya (DMU 13) Bank efficiency (0.9969) having more scope to reach an efficient DMU as this bank need to improve just 0.0031 of efficiency.

Table-I shows the SBM technical efficiency benchmark (peers) for all the PSBs under CRS method. The peer score represents the weights to construct a linear combinational of the efficient banks to represents an inefficient one. From the peer count of efficient DMUs, Kotak Mahindra (DMU 14), Thamilnad Mercantile (DMU 18) are more used than Federal (DMU 6), J & K (DMU 11), IDBI (DMU 10) and City (DMU 3) as peer. So, using SBM CRS input technical efficiency DEA, the DMU 14, 18 are most efficient than other efficient DMUs 6, 11, 10, 3, 5, 6, 10, 11 and 20 have efficiency score equal to one. Hence, DMU 14, 18 are most efficient and referred DMU for other DMUs.

The above table useful for the evaluation of benchmark to the inefficient DMUs and inefficient DMUs become efficient once they attain the benchmark DMUs performance. Under the CRS result a DMU 14, 18 is the maximum number of cases benchmark to other inefficient DMUs. Hence peer count is the benchmark to identify a DMU is most efficient. The DMUs Dhanalakshmi (DMU 5), IndusInd (DMU 09) and Bandhan (DMU 20) are just efficient banks as its input efficiency score is equal to one but these DMUs are not reference set (peer) with other inefficient DMUs presented in the above table and these DMUs are peer themselves.

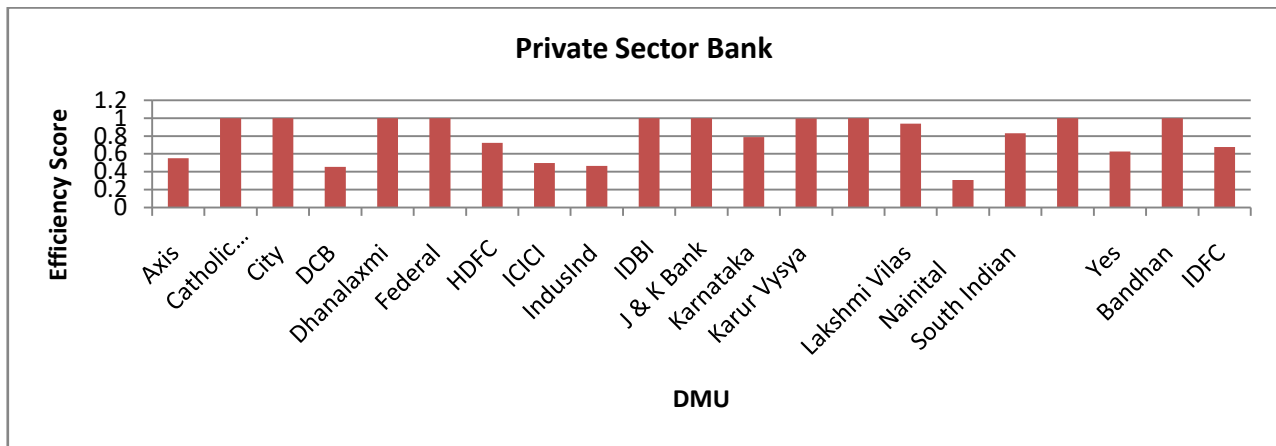


Fig – 1. SBM CRS Efficiency score projection of Pvt.SBs using DEA

The above graph represents the efficiency score projection of Pvt.SBs under SBM CRS approach. The peaks of the bars on X, Y axis are based on efficiency scores of identified DMUs in DEA model. From the graph we conclude that there are 10 DMUs are touch the high peaks, it represents these DMUs are efficient. The poor performer

DMUs from the graph are Nainital, DCB, IndusInd, ICICI and Axis banks in the sequence. A DMU having higher scope to become an efficient is Lakshmi Vilas Bank as this bank DMU peak is nearer to efficiency score 1.

C. Output of Slack Based Measure of Efficiency

Table-II: Efficiency Benchmark of SBM VRS method of DMUs using DEA

S. No	DMU	Score	Rank	VRS Benchmark (Lambda)	VRS Peer
1	Axis	0.6841	18	HDFC (0.441); IDBI (0.532); Kotak Mahindra (0.03)	-
2	Catholic Syrian Bank Ltd.	1	1	Catholic Syrian	2
3	City	1	1	City	2
4	DCB	0.5009	21	J And K Bank (0.164); Kotak Mahindra (0.002); Thailand Mercantile (0.038);CatholicCyrian (0.796)	-
5	Dhanalaxmi	1	1	Dhanalaxmi	0
6	Federal	1	1	Federal	3
7	HDFC	1	1	HDFC	3
8	ICICI	0.6475	19	HDFC (0.60); IDBI (0.381); Kotak Mahindra (0.034);	-
9	IndusInd	0.5295	20	Federal (0.02); IDBI (0.672); TamilnadMercantile(0.368)	-
10	IDBI	1	1	IDBI	5
11	J & K Bank	1	1	J & K Bank	4
12	Karnataka	0.7912	15	City (0.465); Federal (0.055); J & K (0.48)	-
13	Karur Vysya	1	1	Karur Vysya	0
14	Kotak Mahindra	1	1	Kotak Mahindra	4
15	Lakshmi Vilas	0.9548	13	Catholic Syrian (0.763); J & K Bank (0.204); Tamil Mercantile (0.033)	-
16	Nainital	1	1	IDBI (0.025)	0
17	South Indian	0.8429	14	City (0.485); Federal (0.345); J & K (0.17)	0
18	Thamilnad Mercantile	1	1	Thamilnad Mercantile	4
19	Yes	0.6973	16	HDFC (0.028); IDBI (0.932); Kotak Mahindra (0.08)	-
20	Bandhan	1	1	Bandhan	0
21	IDFC	0.6959	17	IDBI (0.604); Thamilnad (0.396)	-

The private sector banks are exposed to a common production frontier on production possibility set of DEA. The objective of SBM BCC is Variable Returns-to-Scale (VCR) and it assumed to be decrease, increase and constant. The largest θ (SBM VRS) efficiency score 1 imply DMU(s) is technically efficient and the rest of the DMUs are inefficient whose efficiency score is less than 1.

The output of SBM VRS approach from Pvt.SBs exhibits the results as the efficient banks Catholic Syrian (DMU 2), City (DMU 3), Dhanalakshmi (DMU 5), Federal (DMU 6), HDFC (DMU 7), IDBI (DMU 10), J & K (DMU 11), Karur Vysya (DMU 13), Kotak Mahindra (DMU 14), Nainital (DMU 16), Tamil Mercantile (DMU 18) and Bandhan (DMU 20) Bank. These banks are efficient under Constant returns to scale on frontier line of production possibility set and these are the benchmark to the inefficient banks. We see that under SBM CRs results out of 21 DMUs, 12 DMUs are efficient and 9 are the inefficient DMUs.

The result from the SBM CRS approach, the banks whose efficiency is decline under constant returns to scale is DCB (DMU 16) bank whose efficiency score is 0.5009. This DMU technically 50% need to improve to become an efficient DMU. Similarly, the other inefficient DMUs under SBM CRS approach are IndusInd (DMU 9) (0.5295), ICICI (DMU 8) (0.6475), Axis (DMU 1) (0.6841) and IDFC (DMU 21) bank. A remarkable thing from the efficiency score is that the largest commercial bank ICICI showed poor performance as its efficiency score is decline at SBM CRS. To become an efficient DMU, the ICICI need to recover 35.25% of

its efficiency score without increasing its input. Under this model, Lakshmi Vilas (DMU 15) Bank efficiency (0.9548) having more scope to reach an efficient DMU as this bank need to improve just 0.0452 of efficiency.

Table-II shows the SBM technical efficiency benchmark (peers) for all the PSBs under VRS method. The peer score represents the weights to construct a linear combination of the efficient banks to represents an inefficient one. From the peer count of efficient DMUs, IDBI (DMU10) are more used than Kotak Mahindra (DMU 14), Tamilnad Mercantile (DMU 18), Federal (DMU 6), J & K (DMU 11) and HDFC (DMU 7) as peer. So, using SBM VRS input technical efficiency DEA, the DMU 10 is most efficient than other efficient DMUs 11, 14, 18, 6, 7, 2, 3, 5, 13, 16, 17 and 20 have efficiency score equal to one. Hence, DMU 10 is most efficient and referred DMU for other DMUs.

The above table useful for the evaluation of benchmark to the inefficient DMUs and inefficient DMUs become efficient once they attain the benchmark DMUs performance. Under the VRS result a DMU 10 is the maximum number of cases benchmark to other inefficient DMUs. Hence peer count is the benchmark to identify a DMU is most efficient. The DMUs Dhanalakshmi (DMU 5), Karur Vysya (DMU 13), Nainital (DMU 16), South India (DMU 17) and Bandhan (DMU 20) are just efficient banks as its input efficiency score is equal to one but these DMUs are not reference set (peer) with other inefficient DMUs presented in the above table and these DMUs are peer themselves.

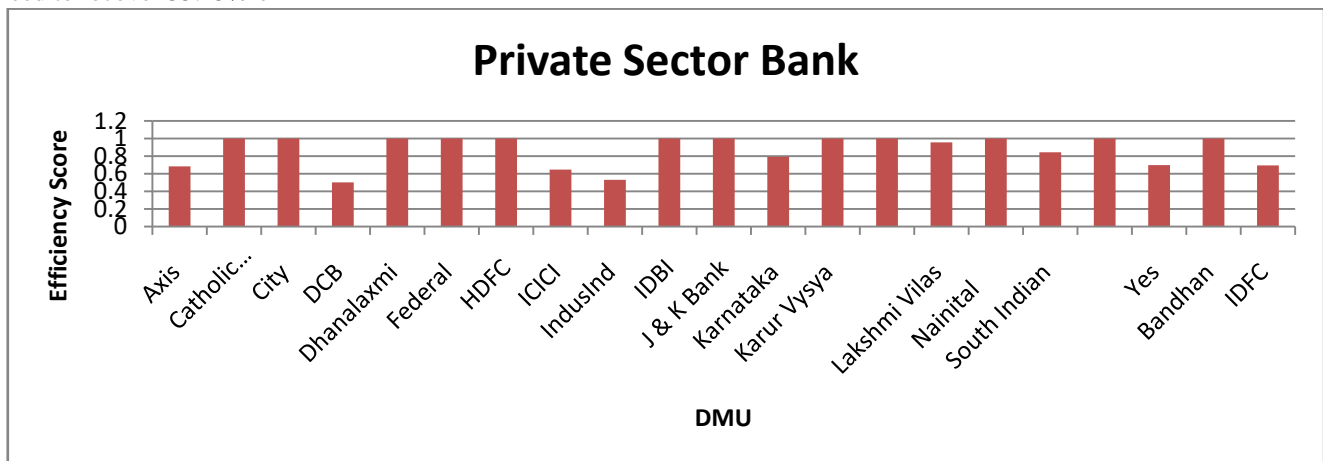


Fig – 2: SBM VRS Efficiency score projection of Pvt.SBs using DEA

The above graph represents the efficiency score projection of Pvt.SBs under SBM VRS approach. The peaks of the bars on X, Y axis are based on efficiency scores of identified DMUs in DEA model. From the graph we conclude that there are 12 DMUs are touch the high peaks, it represents these DMUs are efficient. The poor performer DMUs from the graph are Nainital, DCB, IndusInd, ICICI and Axis banks in the sequence. A DMU having higher scope to become an efficient is Lakshmi Vilas Bank as this bank DMU peak is nearer to efficiency score 1.

Table-III: Descriptive efficiency statistics of SBM CRS, VRS approach of Pvt. SBs

***	CRS	VRS
Average	0.8031	0.8735
Max	1	1
Min	0.3098	0.5009
St Dev	0.2336	0.1751

The above descriptive statistics represents the efficiency basic information obtained from SBM under CRS, VRS model. The above results represents the mean difference between SBM CRS and VRS approach and it is found that the variation in mean efficiency scores. The standard deviation of efficiency represents the variability within the model and it exhibits the less variability found in VRS model. The overall result from the above two model is significant change in two approaches. This significance change of two model statistically proved by Wilcoxon Mann Whitney U-test.

IV. STATISTICAL ANALYSIS OF SIGNIFICANCE AMONG TWO APPROACHES

The DEA is the non-parametric approach used to measure the efficiency of identified DMUs at input, output variables. This approach is free from the assumption Normality and it is depends on the ranking of the efficiency scores of given DMUs. To test the significance among identified two models of DEA, the best non-parametric approach is Wilcoxon Mann Whitney U - test. The aim of this non-parametric approach is test the significance of efficiency among identified model using its ranks of the efficiencies.

Test Statistic

The U statistic is computed as shown in the following formula:

$$U = \frac{s \frac{m(m+n+1)}{2}}{\sqrt{\frac{mn(m+n+1)}{12}}}$$

U = 3.0594

Using the P-value approach: The p-value is p = 0.00222

Since p=0.00222 < 0.05, it is concluded that the null hypothesis is rejected.

Conclusion

It is concluded that the null hypothesis Ho is rejected. Therefore, there is an enough evidence to claim that efficiency score of two approaches differ significantly.

V. CONCLUSION

The Data Envelopment Analysis is the linear mathematical programming which deals with optimizing the optimum solution of Decision Making Units of identified input, output variables. The DEA is the benchmark procedure used to identify efficient DMUs from the variables and these variables are benchmark to the inefficient DMUs. The DEA is the non-parametric approach used to measure the efficiency DMUs at selected input and output variables. The conclusion of this article is based on SBM CCR, BCC models in DEA. Using BCC model input, output variables are selected by Step-wise method. The variables Borrowings, Number of Employees are identified as input variables and Investment, Loans & Advances and Deposits are output variables. The analysis of identified input, output variables can do by SBM CCR, BCC models. The peer count (reference set) is the procedure to identify the benchmark (outperform) variables from the given set of variables.

The result from the SBM CRS approach is the DMUs Kotak Mahindra (DMU 14), Tamil Mercantile (DMU 18), Federal (DMU 6) and Jammu & Kashmir (DMU 11) are the best performer in the sequence and worst performer banks are Dhanalakshmi (DMU 5), IndusInd (DMU 9) and Bandhan (DMU 20) Banks. From the SBM VRS approach, the outperformer banks are IDBI (DMU 10), Kotak Mahindra (DMU 14), Tamil Mercantile (DMU 18), Federal (DMU 6) and Jammu & Kashmir (DMU 11) Banks and performance decline DMUs are Dhanalakshmi (DMU 5), Nainital (DMU16) and Bandhan (DMU 20) Banks. The overall best performe banks from the above said models are Kotak Mahindra, Tamil Mercantile, Federal, IDBI and Jammu & Kashmir Banks.

REFERENCES

1. Andersen P, Petersen NC. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, 39(10), 1261-4.pp. 1078-92. <https://doi.org/10.1108/09600030810915152>
2. Banker, R.D., Charnes, A. and A. Cooper WW. (1984). Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. *Management Science*, 30(9). <https://dpo.org/10.1287/mnsc.30.9.1078>
3. Charnes A, Cooper WW, Rhodes E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2: 429-44. *Management Science*, 27, pp.668-697. <https://doi.org/10.1515/apjri-2018-0027>
4. Charns, W. W. Cooper and E. Rhodes. (1981). Evaluating Program and Managerial Efficiency. An Application of Data Envelopment Analysis to program Follow Through, pp-654-15. <https://doi.org/10.1287/mnsc.27.6.668>.
5. Morita H, Hirokawa K & Zhu J. (2005). A slack-Based Measure of Efficiency in Context-Dependent Data Envelopment Analysis. *Omega*, 33, 357-362. <https://doi.org/10.1016/j.omega.2004.06.001>
6. P. Andersen and N.C. Petersen. (1993). Detecting Influential Observations in Data Envelopment Analysis, *Journal of Productivity Analysis*, 6, pp. 27-46. https://doi.org/10.1007/0-387-29122-9_10
7. Raju Nellutla, V. V Haragopal (2017) "Technical efficiency Management wise Schools in Secondary School Examinations of Andhra Pradesh State by CCR Model" *IOSR Journal of Mathematics*, Vol-13, issue1, pp.1-8.
8. Subramanyam T (2006). Selection of Input-Output Variables in Data Envelopment Analysis – Indian commercial Banks, *International Journal of Computer & Mathematical Sciences*, 5, 2347-8527.
9. Tone K. (1997). A slack-based Measure of Efficiency in Data Envelopment Analysis, *Research Reports*, Graduate School of Policy Science, Saitama University and subsequently published in *European Journal of Operational Research*, 130, pp.498-509. <https://doi.org/10.1007/0-387-29122-9>
10. Tone K. (2001). A Slack-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, 130, 498-509. [https://doi.org/10.1016/S0377-2217\(99\)00407-5](https://doi.org/10.1016/S0377-2217(99)00407-5)
11. Tone K. A. (2002). Slack-based measure of super-efficiency in data envelopment analysis. *European Journal of Operational Research*, 143 (1):32-41. <https://doi.org/10.1057/jors/2013.40>
12. Tone K, Tsutsui M. (2009). Network DEA: A slacks-based measure approach, *European Journal of Operational Research*, 197, 243-252. <http://doi.org/10.1016/j.ejor.2008.05.027>
13. Tone K, Tsutsui M. (2010). Dydynamic DEA: A slacks-based measure approach, *Omega*, 38, 145-156. <https://doi.org/10.1016/j.cie.2015.01.008>

AUTHOR PROFILE



B. Vittal, working as Assistant professor in statistics, Department of Computer Science and Engineering, CVR College of Engineering, JNTUH, Hyderabad, India. His area of research includes Data Envelopment Analysis, Operations Research, and Time Series Analysis.



Dr. Raju Nellutla, working as Associate Professor of Mathematics/ Statistics and Department Coordinator in Humanities and Sciences Department, Guru Nanak Institutions Technical Campus(A), Hyderabad, Telangana, India. His area of research includes Applied Statistics, Advanced Operations Research and Data Envelopment Analysis and Data science.



Dr. M. Krishna Reddy, Working as Professor of Statistics, Department of Computer Science and Engineering, CVR College of Engineering, JNTUH, Hyderabad, India. His area of research includes Time Series Analysis, Neural Network and Data Envelopment Analysis.