Complete Regular Fuzzy Graphs

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***Abstract**: In this paper, some properties of complete degree and complete regular fuzzy graphs are discussed. They are illustrated through various examples. It is proved that every fuzzy graph is an induced subgraph of a complete regular fuzzy graph. The procedure described in the proof is illustrated through an example. Also the complete degree of a vertex in fuzzy graphsformed by the operation Union in terms of the complete degree of vertices in the given fuzzy graphs for some particular cases are obtained. Using them, their complete regular property is studied.*

I. INTRODUCTION

Azriel Rosenfeld was introduced Fuzzy graph theory in 1975 [9]. Bhattacharya [1] gave some remarks on fuzzygraphs.Mordeson.J.N. and Peng.C.S.were introduced Some operations on fuzzy graphs [4]. Zadehintroduce a mathematical frame work to describe the phenomena of uncertainty in real life situation has been suggested in 1965[3].

Research on the theoryof fuzzy sets has been witnessing an exponential growth of both mathematics and in its applications. This rangesfrom traditional mathematical subjects like logic topology, algebra, analysis etc. to pattern recognition, information theory, artificial intelligence, operations research, neural networks and planning etc. Mordeson.J.N. and Peng.C.S were defined by the operations of union, join, Cartesian product and composition on two fuzzy graphs [4]. Nagoorgani.A and Radha. K. was discussed the degree of a vertex in some fuzzygraphs and the Regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operations union,join, Cartesian product and composition [8].

In this paper we study aboutsome properties of complete regular fuzzy graphs. First we go through some basic definitions which can be found in [1-13].

Definition 1.1 ([2]).“A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G : (σ,μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ, i.e., μ(uv) = σ(u)∧σ(v). The underlying crisp graph ofG : (σ,μ) is denoted by G∗ : (V,E), where E ⊆ V ×V .

Definition 1.2 ([2]).G0 : (σ0,μ0) is a fuzzy sub graph or a partial fuzzy sub graph of G : (σ,μ) if σ0⊆ σ and μ0⊆ μ; that is if σ0(u) ≤ σ(u) for every u ∈ V and μ0(uv) ≤ μ(uv) for everyuv∈ E.

Definition 1.3 ([2]). G0 : (σ0,μ0) is a fuzzy spanning sub graph of G : (σ,μ) if σ0 = σ and μ0⊆ μ; that is if σ0(u) = σ(u) for every u ∈ V and μ0(uv) ≤ μ(uv) for every uv∈ E.

Definition 1.4 ([2]). For any fuzzy subset ‘ ν ‘ of V such that ν ⊆ σ, the fuzzy sub graph of G : (σ,μ) induced by ν is the maximal fuzzy sub graph of G : (σ,μ), that has fuzzy vertex set ν and it is the fuzzy sub graph H : (ν,τ) where τ(u,v) = τ(u)∧τ(v)∧μ(u,v) for all u,v in V .

Definition 1.5 ([7]). Let G : (σ,μ) be a fuzzy graph on G∗ : (V,E). The degree of a vertex u is

dG(u) =∑_(u≠ v)▒μ(uv). The minimum degree of G is δ(G) = ∧{dG(v), ∀ v ∈ V} and the maximum degree of G is Δ(G) = ∨{dG(v), ∀ v ∈ V}.

Definition 1.6 ([5]). The order and size of a fuzzy graph G are defined by O(G) =∑_(u∈V)▒〖σ (u)〗) and S(G) =∑_(uv∈E)▒〖μ(uv).〗

Definition 1.7 ([7]). Let G : (σ,μ) be a fuzzy graph on G∗ : (V,E). Then σ is a constant function if and only if thefollowing are equivalent:

1. G is a regular fuzzy graph.
2. G is a Complete regular fuzzy graph”.

Note:“Throughout this paper G_1* : ($σ_1$,$μ_1$) and G_2* : ($σ_2$,$μ_2$) denote two fuzzy graphs with underlying crisp graphs G1*: (V1,E1) and G $_2$*: (V_2,E_2) with |Vi| = pi, i = 1,2. Also dG∗ i (ui) denotes the degree of ui in G∗I”.

Definition 1.8 ([8]). The union of two fuzzy graphs G1 and G2 is defined as a fuzzy graph G = G_1∪ G_2 : ($σ_1$∪ σ $_2$,μ1∪ $μ_2$) on G∗(V,E) where V = V_1 ∪ V_2 and E = E_1∪ E_2 with

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$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u), & if\ u \in V_1 - V_2 \\ \sigma_2(u), & if\ u \in V_2 - V_1 \\ \sigma_1(u)\ V\ \sigma_2(u), & if\ u \in V_2 \cap V_1 \end{cases}$$

$$(\mu_1 \cup \mu_2)(u) = \begin{cases} \mu_1(e), & if\ e \in E_1 - E_2 \\ \mu_2(u), & if\ e \in E_2 - E_1 \\ \mu_1(e)\ V\ \sigma_2 e, & if\ e \in E_2 \cap E_1 \end{cases}$$

Definition 1.9 ([7]). Let G : (σ,μ) be a fuzzy graph. The degree of a vertex u in G is defined by

$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$

Definition 1.10 ([8]).“Let G : (σ,μ) be a fuzzy graph on G∗. The total degree of a vertex u ∈ V is defined by

$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u)$

If each vertex of G has the same complete degree k, then G is said to be a complete regular fuzzy graph of complete degree k or a k- completeregular fuzzy graph.

Notation 1.11. The relation σ1 ≤ μ2 means that σ1(u) ≤ μ2(e) ∀ u ∈ V1 and ∀ e ∈ E2 where σ1 is a fuzzy subset of V1 and μ2 is a fuzzy subset of E2.

Lemma 1.12 ([8]). If G1 : (σ1,μ1) and G2 : (σ2,μ2) are two fuzzy graphs such that σ1 ≤ μ2 then σ2 ≥ μ1.

Definition 1.13 ([7]Total degree of a vertex). Let G : (σ,μ) be a fuzzy graph on G∗ : (V,E). The total degree of a vertex u ∈ V is defined by

$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u)$”.

Example 1.14. Consider the following fuzzy graph G : (σ,μ).

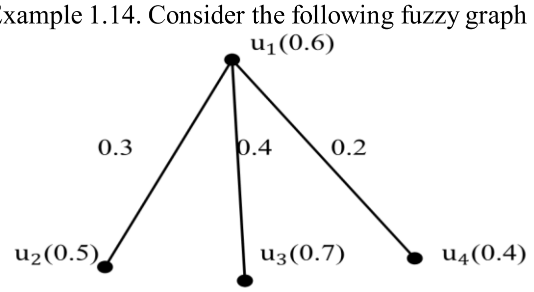

Figure 1.

tdG (u1)=[μ(u1u2)+μ(u1u3)+μ(u1u4)]+s(v)=[0.3 + 0.4 + 0.2 + 0.6]= 1.5

Similarly,

tdG (u2)= 0.8; tdG (u3)= 1.3; tdG (u4)= 0.6

2.Complete Regular Fuzzy Graph

“ Let G : (σ,μ) be a fuzzy graph on G∗ : (V,E). If each vertex in Ghassame total degreek,then Gissaid to be a Complete regular fuzzy graph or k- complete regular fuzzy graph”.

Example :2.1Consider the following fuzzy graph G : (σ,μ)

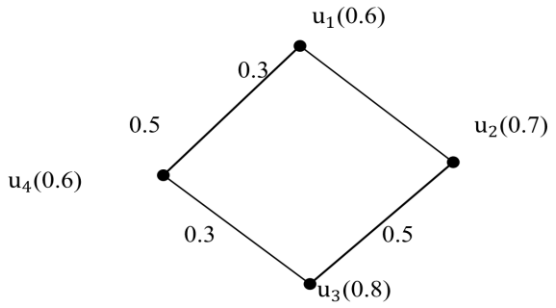

Figure 2.

“ The fuzzy graph in Figure 2 is a 1.2- Complete regular fuzzy graph. Also it is a 0.6-regular fuzzy graph”.

Example 2.2.

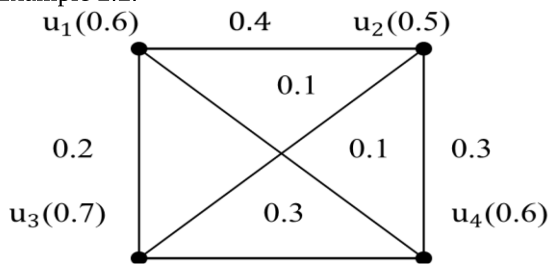

Figure 3.

“ The fuzzy graph in Figure 3 is a 1.3- complete regular fuzzy graph. But it is not a regular fuzzy graph”.

Example 2.3.

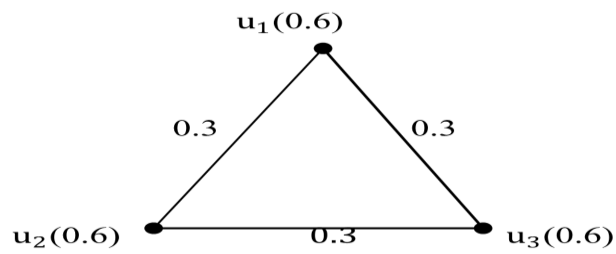

Figure 4.

“ The fuzzy graph in Figure 4 is a 0.8-regular fuzzy graph. But it is not a complete regular fuzzy graph”.

Example 2.4.

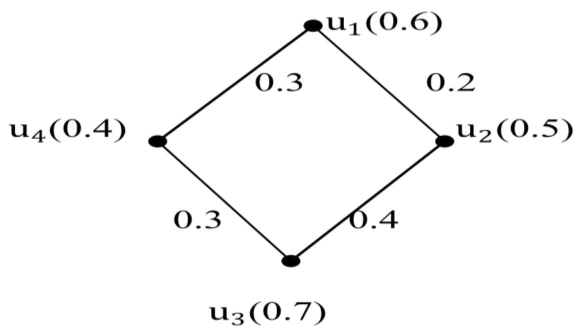

Figure 5.

" The fuzzy graph in fig.5 is neither regular nor complete regular fuzzy graph".

Remark 2.5. From the above examples, it is clear that in general there does not exist any relationship between regular fuzzy graphs and complete regular fuzzy graphs.

Theorem 2.6 ([7]).“Let G : (σ,μ) be a fuzzy graph on G∗ : (V,E) Then σ is a constant function if and only if the following are equivalent :
(1). G is a regular fuzzy graph.
(2). G is a Complete regular fuzzy graph”.

3. Properties of Complete Regular Fuzzy Graphs:
Theorem 3.1.“In any fuzzy graph G, if σ(v) > 0 for every vertex v ∈V , thentd (v) > 0, for every vertex v ∈ V .
Proof. Since σ(v) > 0 for every vertex v ∈V , td (v) > 0, for every vertex v∈ V”

Theorem 3.2.“The maximum total degree of any vertex in a fuzzy graph with p vertices is p.

Proof. For any vertex v,
$td_G(v) = \sum_{uv\in E}\mu(uv)+s(v) = \sum_{uv\in E}1+1$
$= d_G^*(v) +1 = [p-1]+1=p$”

Theorem 3.3.“The total degree of a vertex v is σ (v) if and only if the degree of v is 0.

Proof. The total degree of a vertex v is
$td_G(v)=s(v)$

$\leftrightarrow \sum_{uv\in E}\mu(uv)+s(v) \leftrightarrow \sum_{uv\in E}\mu(uv)=0$

$\leftrightarrow d_G(v) =0$”.

Corollary 3.4.“The total degree of a vertex v is σ (v) for every vertex v in G if and only if G is a null fuzzy graph.

Proof.tdG (v) = s (v), for every vertex , v ∈ V ↔dG (v) = 0, for every vertex
v∈ V ↔G is a null fuzzy graph”.

Theorem 3.5.“Every fuzzy graph is an induced fuzzy subgraph of a Complete regular fuzzy graph.
Proof :Let G : (V;E) be any fuzzy graph with p vertices and q edges.
If G is Complete regular, there is nothing to prove.
Suppose that G is not Complete regular.
LetΔt = max{td(v) / v ∈ V }
Let us prove that G is an induced fuzzy subgraph of a Δt - Complete regular Fuzzy Graph.
Take a copy G0 of G. Take any vertex v with total degree less than Δt
Join it s a copy v0 in G0
Assign min {σ(v),Δt-tdG(v)}as the membership value of the edge vv0
Do this for all vertices with tdG0 (v) <ΔtinGo
Let the resultant fuzzy graph be G1.
For any vertex v with td (v) <Δt

If μ (vvo) = min {σ(v),Δt-tdG(v) } = Δt -tdG(v) Then

tdG1(v)=tdG(v)+μ (vvo)= tdG(v)+Δt - tdG(v) = Δt

And all the vertices which have total degree Δt in G and their copies in G0 will have the same total degree Δt in G1. Also,

tdG1(vo)=tdGo(vo)+μ (vvo)= tdG(v)+μ (vvo)= Δt

If this happens for every vertices with total degree less than Δt in G and their copies in G0, then the procedure stops here.

If for some vertex v with td (v) <Δt,
μ (vvo) = min {σG(v),Δt-tdG(v)}=σG(v)
Then tdG1(v)<Δt and tdG1(v)<Δt

Now, repeat the above procedure for the fuzzy graph G1 and let the resultant fuzzy graph be G_2. If all the vertices in G_2 have total degree Δt, stop.
Otherwise, continue the procedure till the vertices have total degree Δt in the resultant fuzzy graph.

Let n $= \left[max \left\{ \left(\frac{\Delta^t - td_G(v)}{\sigma(v)} \Big/ td_G(v) \right) < \Delta^t \right\} \right]$

Then the procedure stops after n steps with the Δ^t – complete fuzzy graph Gn .
Also G is an induced fuzzy subgraph of Gn . Here the number of vertices in

$G_n = p + p + 2p + 2^2p + \ldots\ldots\ldots + 2^{n-1}$

$==p+p(1+2+2^2+\ldots\ldots.2^{n-1})$

$= p+\left(\frac{2^n-1}{2-1}\right)p$
$=p+(2^n -1)p$
$=2^np$

The number of edges in $G_n = nq+ \sum_{u\in v}\left[\frac{\Delta^t - td_G(v)}{\sigma_G(v)}\right]$”

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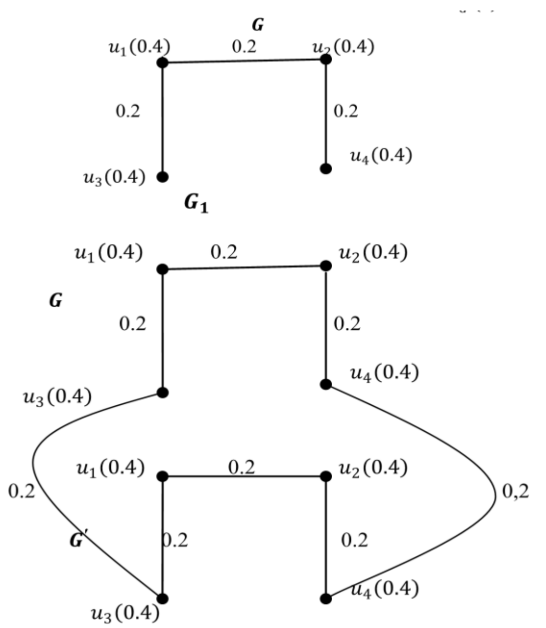

Figure 6.

Theorem 3.6."Let G : (σ,μ) be a fuzzy graph such that both σ and μ are constant partially regular fuzzy graph. Then G is a complete regular fuzzy graph if and only if G is a partially regular fuzzy graph.

Proof : Assume that G is K-complete regular fuzzy graph .
Let μ(uv) = c for all uv∊V and σ(u)=c1 for all u∊V where c and c1 are constants.
Then td_G (u) = dG(u) + σ(u)

$=\sum_{u\epsilon v}\mu(uv)+\sigma(u)$
→ k=cd^*G(u) + c1
→ $d^*{}_G(u) = \frac{k-c_1}{c}$ for all u $\in V$
So G* is regular abd hence G is partially regular fuzzy graph.
Conversly assume that G is a partially regular fuzzy graph
Let G* be r-regular graph

Then
tdG(u) = dG(u) + σ (u)

→ tdG(u) = cd*G(u) + c1

= cr + c1 for all u∊V

So G is Complete regular fuzzy graph.

The following theorem will be helpful in studying various properties of complete regular fuzzy graph.

Theorem 3.7 : Let G1 : (σ_1, μ_1) and G_2 : (σ_2, μ_2) be two fuzzy graphs such that $\sigma_1 = \mu_2$ then $\sigma_1 = \sigma_2$

Proof :Since by the definition of a fuzzy graph, μ_2 (uv)≤σ_2 (u)^ σ2 (v) for all u,v∈V2 we have min $\mu_2 \leq \sigma_2$.

Now $\sigma_1 \leq \mu_2$
→ $\sigma_1 \leq \min \quad \mu_2$
Therefore $\sigma_1 \leq \min \quad \mu_2 \leq \sigma_2$.
→ $\sigma_1 \leq \sigma_2$".

4.Complete Regular Property of Union of Two Fuzzy Graphs

" Let G_1 : (σ_1, μ_1) and G_2 : (σ_2, μ_2) be any two fuzzy graphs underlying crisp Graphs (V_1 and E_1) and (V_2,E_2) respectively.

i) If u∊$V_1 \cup V_2$ and u is arbitrary, then $td_{G1\cup G2} = \begin{cases} td_{G1}(u), u \in V_1 \\ td_{G2}(u), u \in V_2 \end{cases}$

ii) If u ∈ $V_1 \cap V_2$ but no edge incident at u lies in $E_1 \cap E_2$. Then any edge incident at ' u' is either in E_1 or E_2 but not both. Also all these edges will be included in $G_1 \cup G_2$

$td_{G1\cup G2}(u)$ = td_{G1}(u) + td_{G2}(u) -σ_1(u) ^ $\sigma_2(u)$

iii) If u ∈ $V_1 \cap V_2$but no edges incident at 'u' are in E1∩ E2 appear only once in G1∪ G2 andfor this uv

$td_{G1\cup G2}(u)$ = td_{G1}(u) + td_{G2}(u) -σ_1(u) ^ $\sigma_2(u)$-$\sum_{uv\in E1\cap E2}\mu_1$(uv) ^$\mu_2$(uv)

Theorem 4.1: If G1 and G2 are two disjoint k-complete regular fuzzy graph, then $G_1 \cup G_2$ is k-complete regular graph.

Proof : Since G1 and G2 are disjoint fuzzy graphs

$$td_{G1\cup G2} = \begin{cases} td_{G1}(u), u \in V_1 \\ td_{G2}(u), u \in V_2 \end{cases}$$

= k for every u∊ $V_1 \cup V_2$

Therefore $G_1 \cup G_2$ is k-complete Graph"
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