Some Upper Bounds for the Divisor Degree Energy of some Special Graphs

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Abstract: In this paper, we obtain some upper bounds for the divisor degree energy of some special graphs such as total graph, central graph and Q(G).

Keywords : Degree, Energy, Divisor Degree Energy

I. INTRODUCTION

Let us assume that the graph to be a simple graph $G(n,m)$ and d_i be a degree of a vertex v_i.

Gutman [3] was the first to introduce that $E(G) = E(A(G))$, where $A(G)$ is the adjacency matrix of a graph G.

The energy, $E(G)$, of a graph G is defined in [2] to be the sum of the absolute values of its eigenvalues. Hence if $A(G)$ is the adjacency matrix of G, and $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of $A(G)$, then $E(G) = \sum_{i=1}^{n} |\lambda_i|$. With the motivation of Energy, we defined a new energy named divisor degree energy as follows:

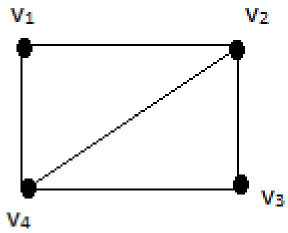
The divisor degree matrix $DD(G)$, a real symmetric matrix with n vertices, is defined in [4] as

$$dd_{ik} = \begin{cases} \left[\dfrac{d_i}{d_k}\right] + \left[\dfrac{d_k}{d_i}\right] & if \;\; v_i, v_k \; are\ adjacent\ and\ d_i \neq d_k \\ 1 & if \;\; v_i, v_k \; are\ adjacent\ and\ d_i = d_k \\ 0 & otherwise \end{cases}$$

where $[x]$ is the integral part of real number x. Then the $n \times n$ real symmetric matrix has its eigenvalues in non-increasing order as $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_n$, where γ_1 is the spectral radius of divisor degree matrix of G.

From the characteristic polynomial $|\gamma I - DD(G)|$, we get the eigenvalues of $DD(G)$. The divisor degree energy (DDE) of a graph is defined as $E_{DD}(G) = \sum_{i=1}^{n} |\gamma_i|$.

Example 1.1. Consider a graph G



The divisor degree matrix of the graph G is

$$DD(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Now, $|\gamma I - DD(G)| = \gamma^4 - 5\gamma^2 - 4\gamma$. Then the eigenvalues of $DD(G)$ are 2.562, - 1.562, -1 and 0 respectively.

Thus, $E_{DD}(G) = 5.124$.

The following definition in [1] is one of the special graph that is needed for the later part of this paper.

For a connected graph G, $Q(G)$ is defined as subdivision of edges of G and joining the edges of new vertices on adjacent edges of G.

II. SOME UPPER BOUNDS FOR THE DIVISOR DEGREE ENERGY OF TOTAL GRAPH OF SOME SPECIAL GRAPHS

In this section, we obtain some upper bounds for the divisor degree energy of total graph of some basic graphs - path graph, cycle graph and star graph.

Theorem 2.1. *Let $T(P_r)$ be a total graph of path graph of order $2r-1$, where r denotes the number of vertices of path graph $P_r, (r > 2)$. Then*

Revised Manuscript Received on December 16, 2019.

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$(i)\, tr(DD(T(P_r)))^2 = 2(4r+1)$

$(ii)\, E_{DD}(T(P_r)) < \sqrt{2(2r-1)(4r+1)}.$

Proof. (i) The divisor degree matrix of $T(P_r)$ is

$$DD(T(P_r)) = \begin{bmatrix} 0 & 2 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & 1 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & & & \ddots & & & \vdots & & & & \ddots & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 2 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 2 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & & & & \ddots & & & \vdots & & & & \ddots & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$$tr(DD(T(P_r)))^2 = 4(r-4) + 4(r-3) + 2\times 5 + 2\times 3 + 2\times 7$$
$$= 2(4r+1).$$

(ii) Using Cauchy-Schwarz inequality,

$$\sum_{i=1}^{2r-1} \gamma_i < \sqrt{2(2r-1)(4r+1)}$$

Therefore, $E_{DD}(T(P_r)) < \sqrt{2(2r-1)(4r+1)}.$

Theorem 2.2. *Let* $T(C_r)$ *be a total graph of cycle graph of order* $2r$, $(r > 2)$. *Then*

$(i)\, tr(DD(T(C_r)))^2 = 8r$

$(ii)\, E_{DD}(T(C_r)) < 4r.$

Proof. (i) The divisor degree matrix of $T(C_r)$ is

$$DD(T(C_r)) = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & 1 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & & & \ddots & & & \vdots & & & & \ddots & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & & & & \ddots & & & \vdots & & & & \ddots & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$$tr(DD(T(C_r)))^2 = 4\times 2r = 8r.$$

(ii) Using Cauchy-Schwarz inequality,

$$\sum_{i=1}^{2r} \gamma_i < \sqrt{2r\times 8r}$$

Therefore, $E_{DD}(T(C_r)) < 4r.$

Theorem 2.3. *Let* $T(K_{1,r-1})$ *be a total graph of star graph of order* $2r-1$. *Then*

$$(i)\, tr(DD(T(K_{1,r-1})))^2 = (r-1)\left(2\left[\frac{r}{2}\right]^2 + 2\left[\frac{2(r-1)}{r}\right]^2 + 2r^2 - 3r \right)$$

$$(ii)\, E_{DD}(T(K_{1,r-1})) < \sqrt{(r-1)(2r-1)\left(2\left[\frac{r}{2}\right]^2 + 2\left[\frac{2(r-1)}{r}\right]^2 + 2r^2 - 3r \right)}.$$

Proof. (i) The divisor degree matrix of $T(K_{1,r-1})$ is

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & \left[\frac{2(r-1)}{2}\right] & \left[\frac{r}{2}\right] & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \left[\frac{2(r-1)}{2}\right] & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \left[\frac{2(r-1)}{2}\right] & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \left[\frac{2(r-1)}{2}\right] & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 & \left[\frac{2(r-1)}{2}\right] & 0 & \cdots & \left[\frac{r}{2}\right] \\ \left[\frac{2(r-1)}{2}\right] & \left[\frac{2(r-1)}{2}\right] & \cdots & \left[\frac{2(r-1)}{2}\right] & 0 & \left[\frac{2(r-1)}{r}\right] & \cdots & \left[\frac{2(r-1)}{r}\right] \\ \left[\frac{r}{2}\right] & 0 & \cdots & 0 & \left[\frac{2(r-1)}{r}\right] & 0 & \cdots & 1 \\ 0 & \left[\frac{r}{2}\right] & \cdots & 0 & \left[\frac{2(r-1)}{r}\right] & 1 & \cdots & 1 \\ \vdots & & \ddots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \left[\frac{2(r-1)}{r}\right] & 1 & \cdots & 1 \\ 0 & 0 & \cdots & \left[\frac{r}{2}\right] & \left[\frac{2(r-1)}{r}\right] & 1 & \cdots & 0 \end{bmatrix}$$

$$tr(DD(T(K_{1,r-1})))^2 = (r-1)\left((r-1)^2 + \left[\frac{r}{2}\right]^2 \right)$$
$$+ (r-1)\left(\left[\frac{r}{2}\right]^2 + \left[\frac{2(r-1)}{r}\right]^2 + (r-2) \right)$$
$$+ (r-1)^3 + (r-1)\left[\frac{2(r-1)}{r}\right]^2$$

$$= (r-1)\left(2\left[\frac{r}{2}\right]^2 + 2\left[\frac{2(r-1)}{r}\right]^2 + 2r^2 - 3r \right)$$

(ii) Using Cauchy-Schwarz inequality

$$\sum_{i=1}^{2r-1} \gamma_i < \sqrt{(r-1)(2r-1)\left(2\left[\frac{r}{2}\right]^2 + 2\left[\frac{2(r-1)}{r}\right]^2 + 2r^2 - 3r \right)}$$

Therefore,

$$E_{DD}(T(K_{1,r-1})) < \sqrt{(r-1)(2r-1)\left(2\left[\frac{r}{2}\right]^2 + 2\left[\frac{2(r-1)}{r}\right]^2 + 2r^2 - 3r\right)}.$$

III. SOME UPPER BOUNDS FOR THE DIVISOR DEGREE ENERGY OF CENTRAL GRAPH OF SOME BASIC GRAPHS

In this division, we obtain some upper bounds for the divisor degree energy of central graph of some basic graphs - path graph, cycle graph and star graph.

Theorem 3.1. *Let* $C(P_r)$ *be central graph of path graph of order* $2r-1, (r>2)$. *Then*

$$(i)\, tr(DD(C(P_r)))^2 = (r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)$$

$$(ii)\, E_{DD}(C(P_r)) < \sqrt{(2r-1)(r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)}.$$

Proof. (i) The divisor degree matrix of $C(P_r)$ is

$$\begin{bmatrix} 0 & 0 & \cdots & 1 & \left[\frac{r-1}{2}\right] & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 & \left[\frac{r-1}{2}\right] & \left[\frac{r-1}{2}\right] & \cdots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & \cdots & 0 & 0 & 0 & \cdots & \left[\frac{r-1}{2}\right] \\ \left[\frac{r-1}{2}\right] & \left[\frac{r-1}{2}\right] & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \left[\frac{r-1}{2}\right] & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \left[\frac{r-1}{2}\right] & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$tr(DD(C(P_r)))^2 = 2\left(r-2+\left[\frac{r-1}{2}\right]^2\right) + (r-2)\left(r-3+2\left[\frac{r-1}{2}\right]^2\right) + 2(r-1)\left(2\left[\frac{r-1}{2}\right]^2\right)$$

$$= (r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)$$

(ii) Using Cauchy-Schwarz inequality

$$\sum_{i=1}^{2r-1}\gamma_i < \sqrt{(2r-1)(r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)}$$

Therefore,

$$E_{DD}(C(P_r)) < \sqrt{(2r-1)(r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)}.$$

Theorem 3.2. *Let* $C(C_r)$ *be central graph of cycle graph of order* $2r\,(r>3)$. *Then*

$$(i)\, tr(DD(C(C_r)))^2 = r\left(r-3+4\left[\frac{r-1}{2}\right]^2\right)$$

$$(ii)\, E_{DD}(C(C_r)) < r\sqrt{2\left(r-3+4\left[\frac{r-1}{2}\right]^2\right)}.$$

Proof. (i) The divisor degree matrix of $C(C_r)$ is

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & \left[\frac{r-1}{2}\right] & 0 & \cdots & \left[\frac{r-1}{2}\right] \\ 0 & 0 & \cdots & 1 & \left[\frac{r-1}{2}\right] & \left[\frac{r-1}{2}\right] & \cdots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & \left[\frac{r-1}{2}\right] \\ \left[\frac{r-1}{2}\right] & \left[\frac{r-1}{2}\right] & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \left[\frac{r-1}{2}\right] & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ \left[\frac{r-1}{2}\right] & 0 & \cdots & \left[\frac{r-1}{2}\right] & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$tr(DD(C(C_r)))^2 = r\left(r-3+2\left[\frac{r-1}{2}\right]^2\right) + r\left(2\left[\frac{r-1}{2}\right]^2\right)$$

$$= r\left(r-3+4\left[\frac{r-1}{2}\right]^2\right)$$

(ii) Using Cauchy-Schwarz inequality

$$\sum_{i=1}^{2r}\gamma_i < \sqrt{2r^2\left(r-3+4\left[\frac{r-1}{2}\right]^2\right)}$$

Therefore,

$$E_{DD}(C(C_r)) < r\sqrt{2\left(r-3+4\left[\frac{r-1}{2}\right]^2\right)}.$$

Theorem 3.3. *Let* $C(K_{1,r-1})$ *be central graph of star graph of order* $2r-1\,(r>2)$. *Then*

$$(i)\, tr(DD(C(K_{1,r-1})))^2 = (r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)$$

$$(ii)\, E_{DD}(C(K_{1,r-1})) < \sqrt{(2r-1)(r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)}.$$

Proof. (i) The divisor degree matrix of $C(K_{1,r-1})$ is

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & \left[\frac{r-1}{2}\right] & \cdots & \left[\frac{r-1}{2}\right] \\ 0 & 0 & \cdots & 1 & \left[\frac{r-1}{2}\right] & \left[\frac{r-1}{2}\right] & \cdots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & \cdots & 0 & 0 & 0 & \cdots & \left[\frac{r-1}{2}\right] \\ \left[\frac{r-1}{2}\right] & \left[\frac{r-1}{2}\right] & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \left[\frac{r-1}{2}\right] & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ \left[\frac{r-1}{2}\right] & 0 & \cdots & \left[\frac{r-1}{2}\right] & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\begin{aligned} tr(DD(C(K_{1,r-1})))^2 &= (r-1)\left[\frac{r-1}{2}\right]^2 + (r-1)(r+2) \\ &\quad + 2(r-1)\left[\frac{r-1}{2}\right]^2 \\ &= (r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right) \end{aligned}$$

(ii) Using Cauchy-Schwarz inequality

$$\sum_{i=1}^{2r-1} \gamma_i < \sqrt{(2r-1)(r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)}$$

Therefore,

$$E_{DD}(C(K_{1,r-1})) < \sqrt{(2r-1)(r-1)\left(r-2+4\left[\frac{r-1}{2}\right]^2\right)}.$$

IV. SOME UPPER BOUNDS FOR THE DIVISOR DEGREE ENERGY OF Q(G) OF SOME BASIC GRAPHS

In this part, we obtain some upper bounds for the divisor degree energy of $Q(G)$ of some basic graphs - path graph, cycle graph and star graph.

Theorem 4.1. *Let* $Q(P_r)$ *be a graph of order* $2r-1$, $(r>2)$. *Then* $(i)\, tr(DD(Q(P_r)))^2 = 6(3r-2)$

$(ii)\, E_{DD}(Q(P_r)) < \sqrt{6(2r-1)(3r-2)}.$

Proof. (i) The divisor degree matrix of $Q(P_r)$ is

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 3 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 2 & 2 & \cdots & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots & \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 2 & 2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 3 \\ 3 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 2 & 2 & \cdots & 0 & 0 & 0 & 1 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 2 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots & \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & \cdots & 2 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 2 & 2 & 0 & 0 & 0 & \cdots & 1 & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 3 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} tr(DD(Q(P_r)))^2 &= 2\times 9 + 2\times 5 + 2\times 11 + (r-4)\times 8 \\ &\quad + (r-3)\times 10 \\ &= 6(3r-2). \end{aligned}$$

(ii) Using Cauchy-Schwarz inequality,

$$\sum_{i=1}^{2r-1} \gamma_i < \sqrt{6(2r-1)(3r-2)}$$

Therefore, $E_{DD}(T(P_r)) < \sqrt{6(2r-1)(3r-2)}.$

Theorem 4.2. *Let* $Q(C_r)$ *be a graph of order* $2r$. *Then*

$(i)\, tr(DD(Q(C_r)))^2 = 52r$

$(ii)\, E_{DD}(Q(C_r)) < 2r\sqrt{26}.$

Proof. (i) The divisor degree matrix of $Q(C_r)$ is

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 2 & 0 & \cdots & 0 & 2 \\ 0 & 0 & \cdots & 0 & 0 & 2 & 2 & \cdots & 0 & 0 \\ \vdots & & \ddots & & & \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 2 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 2 & 2 \\ 2 & 2 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 & 1 \\ 0 & 2 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 2 & 2 & 0 & 0 & \cdots & 0 & 1 \\ 2 & 0 & \cdots & 0 & 2 & 1 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

$$tr(DD(Q(C_r)))^2 = 16\times r + 36\times r = 52p$$

(ii) Using Cauchy-Schwarz inequality,

$$\sum_{i=1}^{2r} \gamma_i < \sqrt{2r\times 52r}$$

Therefore, $E_{DD}(Q(C_r)) < 2r\sqrt{26}.$

Theorem 4.3. *Let* $Q(K_{1,r-1})$ *be a graph of order* $2r-1$, $(r>2)$. *Then*

$$(i)\, tr(DD(Q(K_{1,r-1})))^2 = r(r-1)(2r+1)$$

$$(ii)\, E_{DD}(Q(K_{1,r-1})) < \sqrt{r(r-1)(4r^2-1)}.$$

Proof. (i) The divisor degree matrix of $Q(K_{1,r-1})$ is

$$\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & 0 & r & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & r & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & r & \cdots & 0 & 0 \\
\vdots & & & \ddots & & & \vdots & \vdots & & & \ddots & & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & r & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & r \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 1 & 1 & \cdots & 1 & 1 \\
r & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & 1 & \cdots & 1 & 1 \\
0 & r & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & 0 & \cdots & 1 & 1 \\
0 & 0 & r & \cdots & 0 & 0 & 1 & 1 & 1 & 0 & \cdots & 1 & 1 \\
\vdots & & & \ddots & & & \vdots & \vdots & & & \ddots & & \vdots \\
0 & 0 & 0 & \cdots & r & 0 & 1 & 1 & 1 & 1 & \cdots & 0 & 1 \\
0 & 0 & 0 & \cdots & 0 & r & 1 & 1 & 1 & 1 & \cdots & 1 & 0
\end{bmatrix}$$

$$tr(DD(Q(K_{1,r-1})))^2 = (r-1)\times r^2 + (r-1)\times(r^2+r-1)$$

$$= r(r-1)(2r+1)$$

(ii) Using Cauchy-Schwarz inequality,

$$\sum_{i=1}^{2r-1} \gamma_i < \sqrt{r(r-1)(4r^2-1)}$$

Therefore, $E_{DD}(Q(K_{1,r-1})) < \sqrt{r(r-1)(4r^2-1)}.$

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