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A Host- Mortal Commensal Species Pair With Limited Resources- A Numerical Study

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Abstract

The present paper deals with the numerical study on the ecological model which comprises the commensal and the host with limited resources. The trajectories of this model with multiple constraints are illustrated. The wide range values are considered for the parameters in this model. The dominance reversal time of the host species over the commensal species and vice versa are traced. The interactions between the species are identified.

Keywords*: Non-linear system, Host species, Commensal species, Carrying capacity, Dominance reversal time,* ***AMS classificate****s: 92D25, 92D40*

1. Introduction

Mathematical modeling is having a vast and valuable scope in various fields for investigating better solutions for innovative problems. It is not only used in natural sciences but also plays a vital role in other disciplines like engineering sciences and social sciences. It is mainly intended to study the influencing factors with different components. In most of the cases, the implementation of scientific methods will be depending upon the construction of the mathematical model keeping in view considered situations with experimental measurements. Certain established mathematical techniques have been utilized to solve models effectively and efficiently. Even though it is a multiplexed complexity, computer and simulation techniques are very helpful to get accurate results. Sometimes it is essential to apply direct simulation on the current situation without inventing any model for getting the consistency and possibility of the investigation. Many interactions were discussed by many mathematicians like H. I. Freedman [7-8], G. F. Simmons [6], J. N. Kapur [9], N. Ch. Pattabhi Ramacharyulu and Lakshmi Narayan [10] among the species like Mutualism, Neutralism, Ammensalism, Commensalism, prey-predators, competition etc. In its literature, it explicates continuous and discrete models. Several works have been devoted to investigate these models regarding periodicity, global stability, boundedness and others features. K. V. L. N. Acharyulu and N. Ch. Pattabhi Ramacharyulu [1-5] investigated various models of Ammensalism. Phani Kumar and Seshagiri Rao [11-16] concentrated on Ecological Commensalism.

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The main aim of this paper is to examine a special mathematical model of ecological commensalism with the help of classical method of 4th order Runge-Kutta method. This model contains a Host-Commensal species pair with limited resources in nature. The model is characterized by a couple of first order non linear ordinary differential equations. The effects of change regarding the growth, balanced and mortal coefficients of the commensal species over the host species are identified by fixing other parameters. Sustainability of the ecological relation between the species is discussed. The dominance reversal time is found in all possible cases. The trajectories of this model are drawn and the conclusions are given.

Nomenclature

$N_1(t)$: The population of the commensal (S_1) at time t.

$N_2(t)$: The population of the host (S_2) at time t.

d_1 : The mortal rate of the commensal (S_1).

a_2 : The rate of natural growth of the host (S_2).

a_{11} : The rate of decrease of the commensal (S_1) due to the limitations of its natural resources.

a_{22} : The rate of decrease of the host (S_2) due to the limitations of its natural resources.

a_{12} : The rate of increase of the commensal (S_1) due to the support given by the host (S_2).

$k_1(= a_1 / a_{11})$: The carrying capacity of S_1.

$k_2(= a_2 / a_{22})$: The carrying capacity of S_2.

$c(= a_{12} / a_{11})$: The coefficient of the commensal.

$e_1(= d_1 / a_{11})$: The mortality coefficient of S_1.

t^* : The dominance reversal time.

${t_{g_1}}^*$: The dominance reversal time of the host over the commensal when birth rate is greater than the death rate.

${t_0}^*$: The dominance reversal time of the host over the commensal when death rate is equal to the birth rate.

${t_{e_1}}^*$: The dominance reversal time of the host over the commensal when death rate is greater than the birth rate.

The state variables $N_1(t)$ and $N_2(t)$ as well as all the model parameters d_1, a_2, a_{11}, a_{22}, a_{12}, k_1, e_1, k_2 ,c are assumed to be non-negative constants.

2. Basic Model Equations

The model equations for a two species commensal-host employing the above notations are given by the following system of non-linear coupled ordinary differential equations.

(i). Growth rate equation for the Mortal-Commensal species (S_1)

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$$\frac{dN_1(t)}{dt} = a_{11}\left[-e_1 N_1(t) - N_1^{\,2}(t) + cN_1(t)N_2(t)\right]$$

(1)

(ii). Growth rate equation for the Host species (S_2)

$$\frac{dN_2(t)}{dt} = a_{22}\left[k_2 N_2(t) - N_2^{\,2}(t)\right]$$

(2)

with the initial conditions $N_i(0) = N_{i0} \geq 0, \;\; (i = 1,2).$

(3)

The growth rate equations for **Balanced** (*i.e.* birth rate of the commensal is equal to its death rate) and **Growing** species (*i.e.* birth rate of the commensal is greater than its death rate) can be obtained by taking $e_1 = 0$ and $e_1 = -g_1$ in equation (1).

3. A Numerical Solution of Growth Rate Equations

Numerical solutions of the coupled non-linear basic differential equations (1) and (2) have been computed in the time interval [0, 10] in steps of 0.5, each employing 4th order Runge-Kutta system for a wide range of the model characterizing parameters as below:

The mortal coefficient e_1 of the commensal,

keeping the commensal coefficient $c = 0.8$,

the self inhibition coefficients : $a_{11} = 1,\ a_{22} = 0.15$

and the host carrying capacity $k_2 = 8$ are constants (as shown in the Table-1 and the dominance reversal time of the growing commensal species over the host species is also specified in the Table). The graphical illustrations of the results obtained are shown in Figures 1 to 12.

The illustrations also exhibit the observations on growing and balanced commensal species. These are related to the cases of

(i). Growth rate $g_1 \Rightarrow$ birth rate dominates over the death rate

(ii). The mortality rate $e_1 = 0 \Rightarrow$ death rate balances with birth rate

The results of these two cases: $e_1 = 0$ and $g_1 > 0$ may be realized by replacing $-e_1$ in equations (1) by '0' and $+g_1$ respectively. It may be recalled that $e_1 > 0 \Rightarrow$ the death rate dominates over the birth rate.

Table 1.

S.No.	a_{11}	a_{22}	k_2	c	e_1	g_1	N_{10}	N_{20}	$t_{e_1}^{*}$	t_0^{*}	$t_{g_1}^{*}$
1	**1**	**0.15**	**8**	**0.8**	**0.5**	**0.5**	**1**	**1**	**-**	**-**	**-**
2	**1**	**0.15**	**8**	**0.8**	**1**	**1**	**1**	**1**	**-**	**-**	**-**
3	**1**	**0.15**	**8**	**0.8**	**1.4**	**1.4**	**1**	**1**	**-**	**-**	**1.908**

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4	1	0.15	8	0.8	1.5	1.5	1	1	-	-	2.726
5	1	0.15	8	0.8	1.55	1.55	1	1	-	-	3.501
6	1	0.15	8	0.8	1.8	1.8	1	1	-	-	-
7	1	0.15	8	0.8	2	2	1	1	-	-	-
8	1	0.15	8	0.8	3	3	1	1	-	-	-
9	1	0.15	8	0.8	4	4	1	1	-	-	-
10	1	0.15	8	0.8	5	5	1	1	-	-	-
11	1	0.15	8	0.8	6	6	1	1	-	-	-
12	1	0.15	8	0.8	20	20	1	1	-	-	-

Case: 1

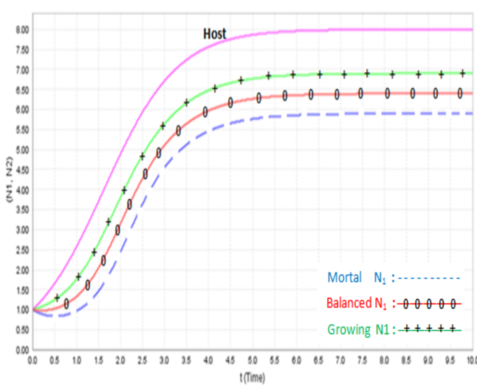


Figure 1

Case: 2

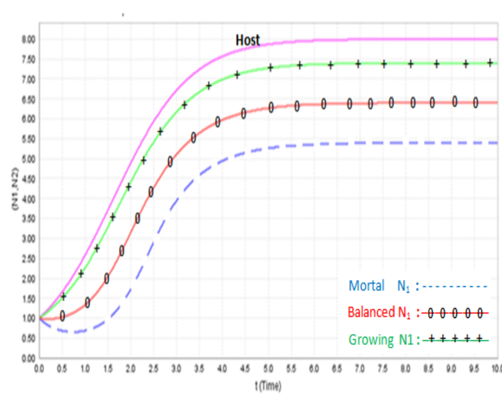
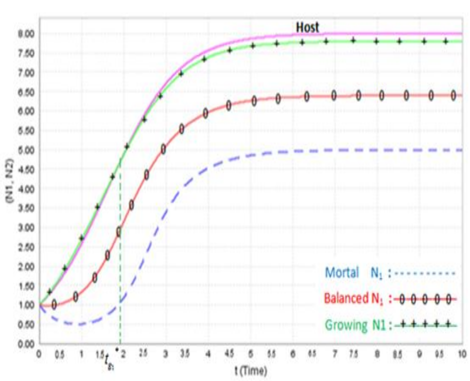


Figure 2

Case: 3



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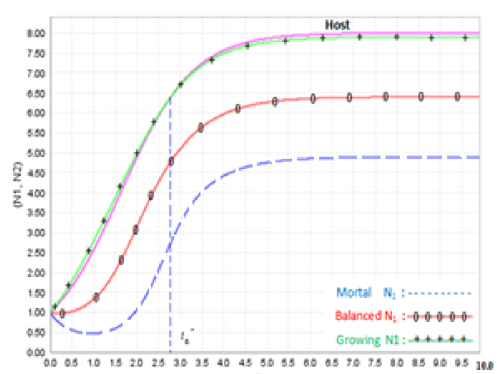

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Figure 3

Case: 5

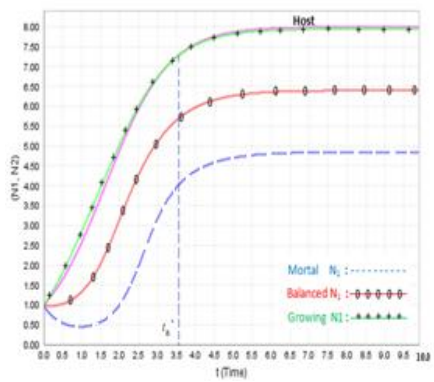

Figure 5

Figure

Case: 6

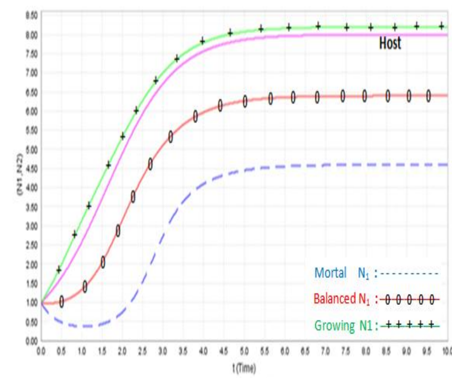

Figure 6

Case 7

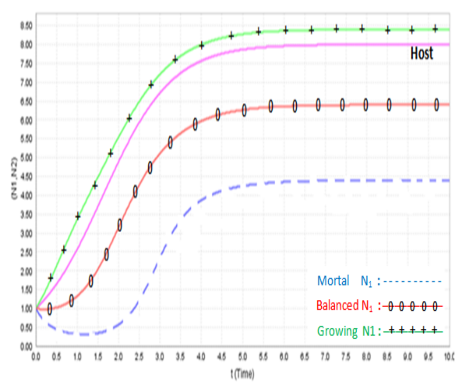

Figure 7

Case 8

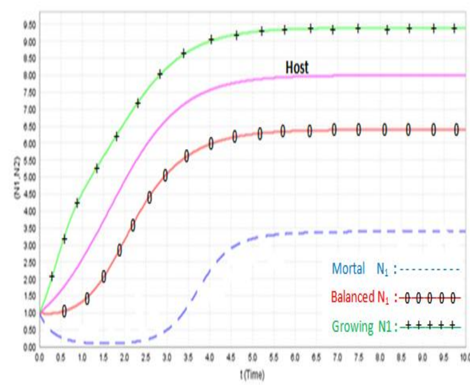

Figure 8

Case 9

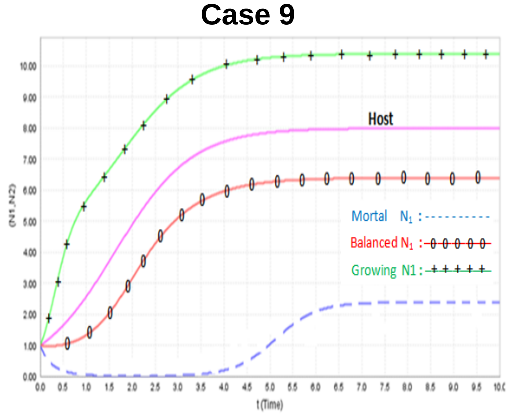

Figure 9

Case 10

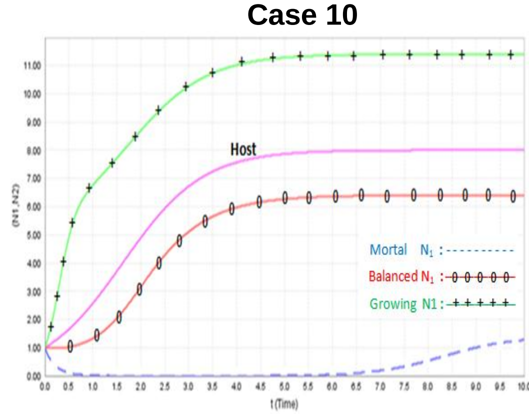

Figure 10

Case 11

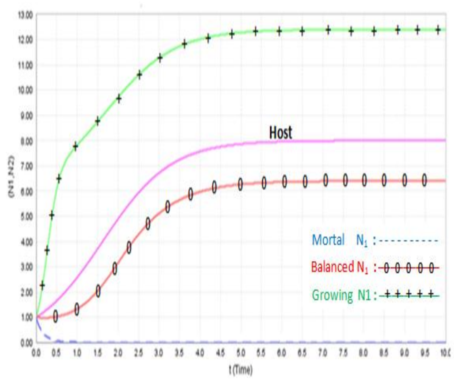

Figure 11

Case 12

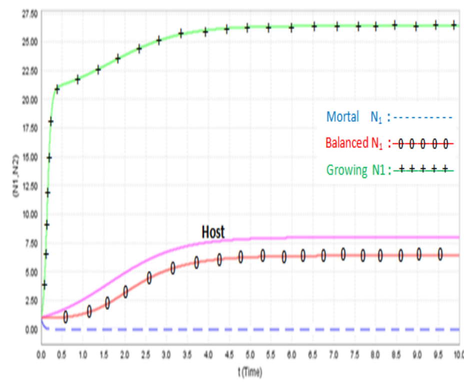

Figure 12

4. Conclusions

(i). Case (1) to **Case (2):** In these two cases the host species out-numbers the commensal species throughout the interval. Also it is observed that both the mortal and balanced commensal decreases initially and then rises whereas both the growing commensal and the host increase steeply. Further, both the species ultimately reach their asymptotic strength numbers (Fig. 1 to Fig. 2).

(ii). Case (3) to **Case (5):** The growing commensal out-numbers the host up to a time (t*) after which the out numbering is reversed. Further there is an appreciable growth rate in both the growing commensal and the host species whereas both the balanced and the mortal commensal initially decrease and then gradually rise to research their respective asymptotic values (Fig. 3 to Fig. 5).

(iii). Case (6) to **Case (9):** In Figures 6 to 9, the trajectories correspond to the values of the mortal commensal coefficient e_1 =1.8, 2, 3, 4 . In these cases 6 to 8 the growing commensal rises steeply and then reaches its asymptotic value. Here there is no appreciable growth rate in both the balanced commensal and the host species where as the mortal commensal decreases before raise.

(iv). Case (10) to **Case (12):** The situation for large values of the mortal commensal coefficient presented in Figures 10 to 12. In these cases growing commensal dominates over the host species throughout. Also it is noticed that there is no appreciable growth rate in both the host and the balanced commensal species where as the growing commensal raises steeply and in the course of time the mortal commensal almost extincts. (Figure 10 to Figure 12).

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