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RAIDS AND IMITATION

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Abstract

Many job changes occur without intervening spells of unemployment. A model is constructed in an attempt to understand this phenomenon. It implies that the best workers are hired away first because, with imperfect information, prices do not fully adjust for quality. Thus, there develops stigma associated with failing to receive outside offers. The force of the stigma, which affects wages, depends upon the likelihood of discovering a worker's ability, the size of the market, and the speed of diffusion of information. In some occupations, it implies that there quickly develop pronounced differences in the treatment of raided and unraided workers. A consequence is a theory of occupational wage dispersion. The Peter Principle--that workers are promoted to a level of incompetence--is a direct implication. The model can be applied to product markets as well to explain the relationship between price and time on the shelf.

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Job changes often occur without spells of unemployment. Highly educated workers, for example, rarely suffer unemployment. A large proportion of their job switches occur only after the new job is secured. These workers, whose skills and ability levels are less homogeneous, differ from less skilled, perhaps more homogeneous workers who are more likely to experience unemployment in the process of changing jobs. Most research has focused on job changes that imply spells of unemployment. Indeed, the primary rationale behind the earliest papers on search theory was to explain unemployment.[1] But if there exists what some refer to as a "dual labor market,"[2] these theories may be most applicable to the secondary workers. This paper attempts to formulate a theory of turnover and wage dynamics that may better describe the primary labor force, defined as those who change jobs without unemployment.[3] In the process, a number of previously unexamined phenomena are explored.

The first task is to understand the relationship between worker quality and turnover. Do markets clear more quickly for the most able workers? Why is it that there is a tendency to try to hire the most able individual, even though his wage rate is higher? It appears that prices do not adjust fully for differences in quality. Buyers constantly seek that diamond in the rough. This also yields the Peter Principle: The best workers are stolen away so those who remain appear incompetent relative to their peers.

The process that is examined makes "stigma" an important feature of labor markets. Because of the information that is produced when workers succeed or fail to receive outside offers, workers who are undesired by outsiders are treated very differently from those who enjoy an active outside market. Thus, stigma, which can be thought of as the consequences of a worker's history of offers and/or employment, is modeled and treated explicitly.

This results in an understanding of the dynamics of worker turnover and wage evolution. Sometimes the patterns may be unusual ones. For example, there may be little action for the first few years of an academic economist's career. Then one job offer triggers others. There is a flurry of activity, wages change rapidly and a job change may or may not occur. After this, a stable period follows, with few offers coming his way again. (Such dynamics are often said to characterize the market for firms where takeover attempts are the equivalent of outside offers.)

The patterns of turnover and wage change can be related in a very simple way to the size of the market, the difficulty associated with learning a worker's ability, and the speed of diffusion of information. For example, when information is difficult to acquire, wages have little dispersion within an occupation, age-earnings profiles are flat, and stigma is unimportant. Further, there is only a very weak relation between ability and tenure. Other relation are easily traced.

A number of implications are derived. Among the more interesting are:

(1) The best workers are raided first. Everyone goes after high quality, highly-priced ones rather than lower quality, lower-priced ones.

(2) Wages differ substantially between workers who receive outside offers and those who do not. This provides implications for wage differentials across occupations and for intra-occupational wage variation.

(3) A corollary is that the importance of stigma depends upon the probability that an outsider recognizes the ability of a given firm's workers and upon the number of buyers. Stigma is not likely to be as pronounced for

assembly line workers as it is for research academicians. As a result, wages are less closely related to ability and age-earnings profiles are flatter for assembly line workers than they are for academicians.

(4) The oldest workers on a given job are the least productive. This paraphrases the Peter Principle[4] and results because the most able of the young workers are bid away.

(5) Workers maximize their expected wealth by working for the firm that is most likely to know their true ability levels. This applies even to low ability workers.

(6) Turnover may exhibit an inverted U-shaped relation to experience for any one worker, even though the aggregate relation is exponentially declining in experience.

Before any implications are derived, it is necessary to construct a simple model and to outline a few basic relations. That is done in the next section.

I. A MODEL

To focus on competition among firms for workers, we begin with a simple model that captures the key features of the effects of informational differences and informed trading. This enables us to examine phenomena such as raids, offer matching and imitation in the labor market.

Suppose that there are two firms, j and k. Firm j hires the worker initially so that given any tie in offers from j and k, the worker remains at j.[5] Firm k initiates a raid by offering $w_k > w_j$. The worker's product at firm j is M, a random variable that, for simplicity, is distributed uniformly between zero and one. It is easy to allow for specific capital but this is ignored throughout.

Information takes the following form: With probability P_j, j learns the true value of M before raiding occurs and with probability $(1 - P_j)$ knows only the ex ante distribution of M at that time. If j learns the truth, then j becomes an "informed" trader. Similarly, with probability P_k (not necessarily equal to P_j), k learns the true value of M before the raiding offer must be made and with probability $(1 - P_k)$, k only knows the ex ante distribution of M.

Public Information:

Raids are selective because the raider is susceptible to "winner's curse."[6] Firm k knows that on average, j lets the worker go when j's assessment of the worker's output falls below w_k. This implies that under many circumstances, any raiding strategy that can succeed results in losses to the raider. Everything depends upon the nature of market information.[7]

To see this, suppose that j always knows what k knows, either because k's strategy is different in all cases so that j can infer what k knows, or because k's information is public. This is characterized by the information in Table 1:

Table 1

Events and Unconditional Probabilities

		j: Informed	j: Uninformed
k	Informed	$P_j P_k + (1 - P_j)P_k = P_k$	0
	Uninformed	$P_j(1 - P_k)$	$(1 - P_j)(1 - P_k)$

The proposition that k can never raid successfully follows trivially from the fact that j has all of k's information, and more. Any strategy that is profitable to k is immediately matched by j. Any unprofitable move is ignored by j so that k steals the workers only when doing so is unprofitable.

Winner's curse occurs whenever k is uninformed. The expected product of the worker is 1/2. But any attempt by k to raid at $w_k \geq 0$ fails or results in losses.

If k is uninformed and j is uninformed (the bottom right cell of Table 1), then j will match any offer up to $E(M) = 1/2$. A raid at any $w_k > 1/2$ is ex ante unprofitable. If j is informed and k is not, then j matches offers when $M > w_k$ so k gets nothing if $M > w_k$. But j declines to match when $w_k > M$. So k only succeeds in stealing the worker when $w_k > M$, and this is a losing strategy. In fact, given that k is uninformed, k's expected gain is

$$(1) \qquad P_j \int_0^{w_k} (M - w_k) f(M) dM$$

$$= P_j \int_0^{w_k} (M - w_k) dM \qquad \text{(since } M \sim U[0, 1])$$

$$= P_j \left(\frac{-w_k^2}{2}\right) < 0$$

for $w_k < 1/2$. (For $w_k > 1/2$, the expected gain is even less at $P_j\left(\frac{-w_k^2}{2}\right) + (1 - P_j)(w_k - 1/2)$.) The expected gain is negative when k is uninformed so no raid occurs when k is uninformed. Further, by assumption, when k is informed, so is j. All raids are unsuccessful. The conclusion is that no raids occur in the absence of private information and firm-specific capital.

Successful Raids:

In order for k to ever attempt a raid on j, at least one of two assumptions must be relaxed: Either there must be specific capital, so that the worker is worth more to k than to j, or k's information must be private. Relaxation of either assumption is sufficient to guarantee that raids sometimes occur. We focus on informational differences.

It is important to consider not only whether a firm is informed about the worker's product, M, but also whether it is informed about the rival's state of knowledge. Both affect the optimal wage strategy. What is essential is that there be some situations when the current employer does not know that his rival is informed. There are a number of justifications for this possibility. First, the worker's claim of an outside offer is not credible. He always has an incentive to claim this if wages rise as a result. Second, k has no incentive to convince j that the offer is genuine because doing so frustrates all raid attempts. On the contrary, k should conceal its attack. Third, the "outside offer" may reflect the worker's value of leisure, which has no bearing on M.[8]

For simplicity, assume that an informed firm is also aware of its rivals' state of knowledge, but that no uninformed firm knows what its rivals know. This allows all the relevant possibilities and saves on tedious and unenlightening discussion. The more general case is described in the appendix.

Under this situation, raids occur only if k is informed. If k is uninformed then it is never profitable to raid. This is easily seen: An uninformed raider encounters either an informed or uninformed firm j. If j is informed, which happens with probability P_j, then no profitable raid can succeed: K only succeeds in stealing those workers for whom $M < w_k$ (otherwise j will match). This results in an expected loss to K of

$$\int_0^{w_k} (M - w_k)dM = \frac{-w_k^2}{2} .$$

If j is also uninformed, which occurs with probability $(1 - P_j)$, only raids at $w_k > w_j$ are successful. Now, in competition, w_j is the maximum wage consistent with zero profits. (This is derived more formally below.) Therefore, if k is uninformed, there is no possibility that an offer of $w_k > w_j$ to a randomly chosen worker can result in profits since K has no informational advantage over j and M is the same at both firms. But $w_k = w_j$ does not result in a successful raid. So, k's profit from raiding when uninformed is zero if j is uninformed, but negative if j is informed. Therefore, k does not raid when uninformed.

Raids do occur, however, when k is informed. Recall that k's move is non-informative. Since w_j is the wage that j offers to a worker about whom j is uninformed, k raids workers at $w_j + \varepsilon$ about whom it knows that $M > w_j$. If j is also informed and j knows that the outside offer is genuine, then the raid fails, because j matches the offer. But with probability $(1 - P_j)$, j is uninformed so that the raiding offer attracts the worker. The expected return to k given that k is informed is

$$(1 - P_j) \int_{w_j}^{1} (M - w_j)dM$$

$$= (1 - P_j)\left[\frac{1}{2} - w_j + \frac{w_j^2}{2}\right] \geq 0 .$$

(It is positive because the minimum value of $\frac{1}{2} - w_j + \frac{w_j^2}{2}$ occurs at $w_j = 1$ where $\frac{1}{2} - w_j + \frac{w_j^2}{2} = 0$ and since $w_j \leq 1$.) So when k is informed that $M > w_j$, a raid occurs.

It might seem that an informed j would behave differently with respect to high-ability workers than with respect to low-ability ones. This is not correct. The informed j could make w_j a function of M. But nothing is gained for workers with $M > w_j$. No $w_j(M) \leq M$ acts as a deterrent to an informed k. No $w_j(M) > M$ is profitable if k is informed and no uninformed k raids. Nothing is gained by conditioning w_j on M, even when j has the information to do so.[9]

Further, risk-neutral workers who do not know their abilities will not sign with any firm that retains the right to reduce wages after observing M. This would result in an expected wage for unraided workers that is less than w_j and since w_j to unraided workers guarantees zero profit, a fixed w_j to all unraided workers dominates.[10]

The results of this section can be summarized: In the absence of any firm-specific capital, raids only occur if raiders have private information and if the raider is informed. For a raid to be profitable, the raider must know the realization of a worker's output and not merely the distribution.

Additionally, raids only occur on workers from the upper part of the ability distribution. Raiders only make profits on those workers, because the current firm pays all workers on which it is uninformed some given wage, w_j. The market has a bias toward underpaying high quality workers.

The Optimal Wage Strategy and Equilibrium:

Now let us derive more formally j's optimal wage strategy and describe the market equilibrium. If both j and k are informed, then j matches any offer by k and $w = M$, the worker is paid his marginal product. Firm j earns zero profit on this worker.

If j is uninformed, but k is informed, k raids. But since j does not believe worker claims of outside offers, k succeeds in stealing the worker at $w_j + \varepsilon$. Again j's profits are zero.

Firm j retains the worker at wage w_j whenever k does not raid. Firm k does not raid either because k is uninformed, or because k is informed that $M < w_j$.

Firm j offers w_j to unraided workers. The goal is to derive w_j. Recall that k only raids when k is informed that $M > w_j$ so j is not be left with a representative sample of workers. Even though j may not be able to verify any one worker's claim of an outside offer (and consequently does not react to it), j knows that workers who actually do leave the firm come from the part of the ability distribution where M exceeds w_j. Again, there are two reasons why a worker remains with firm j: Either the worker has $M > w_j$, but k does not know it, or the worker has $M < w_j$--the worker is an undiscovered star or an already overpaid performer. The probability that $M > w_j$ and k is uninformed is $(1 - w_j)(1 - P_k)$ since M is distributed uniformly between 0 and 1. The expected value of M if this is true is $(1 + w_j)/2$. The probability that $M < w_j$ is w_j (k's information being irrelevant here). The expected value of M if this is true is $w_j/2$. The expected value of a worker who is not stolen away, using the relevant conditional probabilities, is

$$(2) \qquad \frac{(1 - w_j)(1 - P_k)}{(1 - w_j)(1 - P_k) + w_j} \left[\frac{1 + w_j}{2}\right] + \frac{w_j}{(1 - w_j)(1 + P_k) + w_j} [w_j/2] \ .$$

Now, in equilibrium the competitive firm must earn zero profit. Zero profit is already guaranteed for workers who are raided (successfully or unsuccessfully). So in equilibrium, w_j must be set so that zero profits are earned on those workers who are not raided. Thus, from (2),

$$(3) \qquad w_j = \frac{(1 - w_j)(1 - P_k)}{(1 - w_j)(1 - P_k) + w_j} \left[\frac{1 + w_j}{2}\right] + \frac{w_j}{(1 - w_j)(1 - P_k) + w_j} [w_j/2] \ .$$

Equation (2) simply says that j must set w_j so that expected profit on unraided workers is zero, given that only workers with $M > w_j$ are stolen away. Any w_j less than that amount results in positive profits so that another firm can attract all workers. Every w_j greater than that amount results in negative profit.

Solving equation (3) for w_j yields two roots, but one is always negative. The other root is always between 0 and 1/2 and it is given by[11]

$$w_j = \frac{\sqrt{1/(1-P_k)} - 1}{1/(1-P_k) - 1} . \tag{4}$$

The equilibrium can be summarized: Workers come to firm j and are told that all unraided workers are paid

$$w_j = \frac{\sqrt{1/(1 - P_k)} - 1}{1/(1 - P_k) - 1} .$$

If a worker receives an outside offer that j learns is genuine and if j is informed that $M > w_j$, j matches the offer and the worker's wage is driven up to M. (Only workers with $M > w_j$ receive outside offers.) If the worker receives an outside offer and j cannot verify that $M > w_j$, j allows the worker to leave. Zero expected profit is guaranteed.

That $w_j \leq 1/2$ implies that unraided workers are underpaid relative to the population mean. This is because all firms try to steal undiscovered workers whose values exceed their wage. The search for pearls leaves mostly oysters behind so that the output of remaining workers falls short of the population mean.

Raiders' Profits and Costly Information:

As things stand, raiders earn profits. Although raided firms earn zero expected profit, raiders are able to earn profits on those workers whom they succeed in acquiring.

The expected profit on each raided worker is

$$\frac{1}{1 - w_j} \int_{w_j}^{1} (M - w_j)dM$$

(since $\frac{1}{1 - w_j} = \frac{1}{1 - F(w_j)}$). It seems that all firms would prefer to be raiders only. In fact, the more raids attempted, the more profitable the firm.

The reason for this result is that P_k is an exogenous "gift" to outsiders in the current setup. A raider is endowed with a specialized factor--information--P_k of the time and the endowment costs the firm nothing. Firms endowed with such a gift earn rents on the scarce factor.

More realistic, perhaps, is the situation where potential raiders are forced to buy information at some cost. Suppose, for example, that only if a firm bears cost λ does it learn a worker's ability with probability P_k. Otherwise it learns the worker's ability with probability zero. (Think of λ as time spent reading a worker's resumé, which results in useful information P_k of the time.) The expected return on the investment in information is

$$P_k \int_{w_j}^{1} (M - w_j)dM$$

and the cost is λ. The investment is made so long as $\lambda < P_k \int_{w_j}^{1} (M - w_j)dM$. But this still leaves the raider with positive profits, which induces entry into the raiding business. Entry of other raiders reduces the likelihood that any one informed raider acquires the worker. In fact, equilibrium occurs when there are N raiders such that

$$\lambda = \frac{P_k}{N} \int_{w_j}^{1} (M - w_j)dM \ .$$

This not only ensures zero profits for raiders, but also determines the equilibrium number of raiding firms.

Any one firm may act as both raider and employer of new workers, but there is nothing that necessitates tying the two activities together. The determination of whether a firm engages in both types of hiring or just one type is analogous to the determination of the product mix of a multiproduct firm.

Ex Ante Differences in Information:

This section considers what happens when firms have different probabilities of becoming informed. This can arise because firms learn more about their own workers than about others or because some firms are better screeners than others.

What is essential in this model is that there is a difference between ex ante and ex post information. Think of becoming informed about a worker as occurring when reading one of his articles in the AER. If the potential buyer only reads one out of 20 papers per month then $P = .05$. While it is likely that the current employer reads more of his own workers' papers than others' so that $P_j > P_k$, it is still possible that an outsider may read one that the current employer overlooks.

Suppose that before the worker is hired, the two firms have different amounts of information so that $P_j > P_k$. With which firm should the worker sign? Since the wage at the current firm depends upon getting outside offers, should the worker sign with the firm with poorest information? Does this depend upon whether the worker is high or low ability?

These questions are easily answered using eq. (4). If the worker signs with j, then he gets

$$w_j = \frac{\sqrt{1/(1 - P_k)} - 1}{1/(1 - P_k) - 1}$$

if he is unraided, which occurs with probability $(1 - P_k)$. If he is raided and j knows his ability he receives M. This happens with probability P_jP_k. If he is raided and j does not know his ability, then he goes to k at wage w_j + epsilon. This happens with probability $P_k(1 - P_j)$. So expected wage is (ignoring epsilon)

$$E(w)_j = [(1 - P_k) + P_k(1 - P_j)] \frac{\sqrt{1/(1 - P_k)} - 1}{1/(1 - P_k) - 1} + P_jP_kM \,. \tag{5}$$

If he signs with P_k then he has an expected wage of

$$E(w)_k = [(1 - P_j) + P_j(1 - P_k)] \left[\frac{\sqrt{1/(1 - P_j)} - 1}{1/(1 - P_j) - 1}\right] + P_jP_kM \,. \tag{6}$$

When is $E(w)_j > E(w)_k$? The answer is always. It always pays to sign with the firm with the best ex ante information, independent of ability even if the worker knows his own ability.

The proof is straightforward: Assume the opposite. Then $P_j > P_k$ and $E(w_j) < E(w_k)$ so from (5) and (6)

$$\frac{\sqrt{1/(1 - P_k)} - 1}{1/(1 - P_k) - 1} < \frac{\sqrt{1/(1 - P_j)} - 1}{1/(1 - P_j) - 1}$$

which implies $P_j < P_k$. This results in a contradiction.

The intuition follows: First, that ability level is irrelevant is obvious. The worker only gets M when both j and k are informed so the identity of j and k is irrelevant. Second, j knows that the better informed is k, the more likely is j to get stuck with the lowest quality workers. As such, the equilibrium wage that j offers falls with P_k. Since

j experiences less adverse selection, j can (and in equilibrium will) offer a higher wage to unraided workers. Since a worker who is raided is raided at $w_j + \varepsilon$ (unless j is also informed), the worker always benefits from increases in w_j. Thus, he always signs with the firm least susceptible to adverse selection or with the best informed firm, independent of his ability.

The current employer does not pay higher-ability workers higher wages unless an outsider forces him to do so.[12] Even low quality workers benefit from working for the most informed employer. That employer is least likely to be affected by winner's curse and offers the highest equilibrium wage to unraided workers, a wage that is independent of ability levels. It requires two informed buyers to drive the worker's price to M, and their identity relative to the current employer is irrelevant.

Imitation

Most of the interesting phenomena can be analyzed only in a multiperiod, multifirm context. The desire here is to analyze the way that one firm imitates another and to trace the evolution of wages and turnover as it varies with worker ability level.

Think of there being $N + 1$ ex ante identical firms, each of which has a probability P of being informed (for simplicity of notation, assume that P is identical for all N traders). The worker is at one of the $N + 1$ firms, and, as above, call it j. An innovation in this section is that there are two ways to obtain information about a worker: First, one can discover the worker's productivity by oneself--discovery--and this happens with probability P each round. Second, one can infer the worker's productivity--inference--by observing the actions of others.[13] Thus, a raid during the previous round tips off others that the worker's productivity exceeded his previous wage. This often leads to an inverted U-shaped path of raids over the worker's lifetime. Few raids take place initially. Then there is a flurry of activity that dies away rapidly. The shape of the time path of raids depends in interesting ways upon the average levels of information in the market, and upon the number of potential buyers. Additionally, the complete time path of wages for movers and stayers is derived.[14]

The difference between the two types of information is characterized in the following way: Information that the firm obtains by itself with probability P per round can be used in that round. In order to use inferential information, it is necessary to wait one round.[15] The informed trader gets a head start and moves immediately. Those who learn the worker's product by inference, only do so after the informed trader has made his move because it is that move that tips off others.[16]

To derive the complete dynamics, it is useful to consider first the time path of raids on a worker with $M > w_1$ where w_1 is defined as the wage that the current employer pays to unraided workers in round 1. Since, as will be shown below, $w_\tau > w_{\tau-1}$, those workers remain susceptible to raids throughout.

In the first round, there is only one way that a raid can occur: One of the N firms must make a discovery. The probability that this occurs is P for each of the N firms so the expected number of raiding offers in round 1 is

$$R(1)\Big|_{M > w_1} = NP \;. \tag{7}$$

In round 2, things are more complicated. In round 1, there were three relevant possibilities: there were no raids, there was exactly one raid, or there was more than one raid.

Define $\tilde{P} \equiv 1 - P$. With probability $\tilde{P}^N$ there were no raids in round 1 because no outsider was informed. In that case, round 2 is identical to round 1 and there are NP expected raids.

If there was more than one informed firm in round 1, then the equilibrium was that the worker's wage was driven up to M in competition. Consequently, no subsequent raids occur.

The most interesting case is when there was exactly one raid in round 1 and when the current firm, j, was also uninformed. This happens with probability

$$NP\tilde{P}^N .$$

To allow for further generality, allow for a diffusion parameter that may be less than 1: If one raid occurs in round 1 then θN firms are informed via inference at the beginning of round 2.[17] The θN informed firms raid with certainty, whereas the $(1 - \theta)N$ firms can still learn by discovery with probability P. The expected number of raids on the high quality worker in round 2 is

$$(8) \qquad R(2)\Big|_{M > w_1} = \tilde{P}^N NP + NP\tilde{P}^N[\theta N + (1 - \theta)NP]$$
$$= \tilde{P}^N NP[1 + N(P + \theta(1 - P))]$$

Round 3 is similar, but slightly different. If there were any raids in round 1, then the wage rate for the worker was driven up to M by the end of round 2 at the latest. Inference and imitation by others ensures this.[18]

If there were no raids in either round 1 or 2, then round 3 is just like round 1 so the expected number of raids is NP. This occurs with probability $\tilde{P}^{2N}$.

Again the interesting case is when there was exactly one raid in round 2 (and therefore no raids in round 1). That occurs with probability $\tilde{P}^N NP\tilde{P}^N = N\tilde{P}^{2N}P$ and yields expected raids

$$\theta N + (1 - \theta)NP .$$

Thus, the total number of expected raids in round 3 is

$$R(3)\Big|_{M > 2} = \tilde{P}^{2N}NP[1 + N(P + \theta(1 - P))] .$$

In general,

$$(9) \quad R(\tau)\Big|_{M > w_1} = \tilde{P}^{(\tau-1)N}NP(1 + N(P + \theta(1 - P))) \qquad \text{for} \quad \tau \geq 2$$

$$= NP \qquad \text{for} \quad \tau = 1.$$

Using (9), it is clear that

$$(10) \quad R(\tau)\Big|_{M > w_1} - R(\tau - 1)\Big|_{M > w_1} = [\tilde{P}^{(\tau-1)N} - \tilde{P}^{(\tau-2)N}]NP(1 + N(P + \theta(1 - P)) \quad \text{for} \quad \tau \geq 3 .$$

The value of $R(\tau) - R(\tau - 1)$ for $\tau \geq 3$ is always negative because $\tilde{P} < 1$. This implies that beyond round 2, the expected number of raids declines as time progresses.

For $R(2) - R(1)$, the story is different:

$$(11) \quad R(2)\Big|_{M > w_1} - R(1)\Big|_{M > w_1} = NP[(\tilde{P}^N(1 + N(P + \theta(1 - P))) - 1] .$$

This is positive iff $\tilde{P}^N(1 + N(P + \theta(1 - P)) > 1$.

Figure 1 simulates some raiding patterns for different values of N and P.

When $P = 1$, all learning occurs immediately and no raids occur beyond round 1 (see panel 1). For high values of P, the pattern is exponential (see panel 4). But for smaller values of P, an inverted U-shaped pattern results (see panel 3). The reason is that in the first round, the only way that a raid can occur is through independent discovery. But in round 2, raids can occur either by discovery or by inference. Further, all subsequent rounds have a lower expected number of raids because the likelihood of no earlier discovery falls as the number of previous rounds increases.

Also obvious is that an increase in N makes the number of raids in later periods drop off more quickly (compare panel 5 to panel 3). This is like introducing more information into the market.

For very small values of P, raids are unlikely to occur so that life-cycle variation in wages is likely to be smaller--age-earnings profiles are flatter--in occupations where learning about worker ability is very difficult. Stated alternatively, age-earnings profiles for "visible" jobs should be steeper and display more variance across workers than those for less visible jobs at equivalent skill levels. (Competition implies, however, that the expected values of the profiles across the two occupations are the same for similar workers.)

Inverted U-shaped patterns of raids are possible for high quality workers. The likelihood of the inverted U-shape increases with learning by inference relative to discovery: θ, the parameter of diffusion has a clearly positive effect on $R(2)\Big|_{M > w_1} - R(1)\Big|_{M > w_1}$. Also, sufficiently high P eliminates any possibility of inference because all firms learn through discovery and react immediately (compare panel 6 to panel 5).

Stigma

The previous discussion relates only to the high-ability workers. But what of the others in the firm? Related, what can the current firm infer from the failure of some of its workers to receive outside offers and what is the appropriate response in the multiperiod setting? Finally, what is the aggregate pattern of raids and turnover for the entire labor force? These issues are addressed in this section.

First, it is necessary to determine the wage that the current firm offers to all (unraided) workers in round 1. If we allow the spot wage to prevail during each round, then an equation analogous to (3) must hold. The only difference is in the conditional probabilities. The probability that a worker was not raided and that $M > w_1$ is $(1 - w_1)(\tilde{P}^N)$, where w_1 is the wage that the current employer pays all unraided workers in round 1. The probability that he was not raided and $M < w_1$ is simply w_1. So, in equilibrium,

$$w_1 = \frac{(1 - w_1)\tilde{P}^N}{(1 - w_1)\tilde{P}^N + w_1}\left[\frac{1 + w_1}{2}\right] + \frac{w_1}{(1 - w_1)\tilde{P}^N + w_1}[w_1/2] \ . \tag{12}$$

Solving (12) for w_1 yields

$$w_1 = \frac{\sqrt{1/\tilde{P}^N} - 1}{(1/\tilde{P}^N) - 1} \ . \tag{13}$$

Zero expected profits are earned in round 1.

Similar relationships hold for later rounds. In any round τ, workers who remain with the firm at the end of the round fall into one of of two categories: The worker may have $M < w_\tau$ so that no profitable raid is possible. This happens with probability w_τ. Alternatively, the worker has $M > w_\tau$, but no outside firm has discovered this fact in the τ periods. This happens with probability $(1 - w_\tau)\tilde{P}^{N\tau}$. Again, in equilibrium, w_τ must be chosen so that w_τ equals the expected output of workers who are unraided through τ

periods. Thus, the analogue of equilibrium condition (12) in round τ is

$$(14) \qquad w_\tau = \frac{(1 - w_\tau)\tilde{P}^{N\tau}}{(1 - w_\tau)\tilde{P}^{N\tau} + w_\tau} \left[\frac{1 + w_\tau}{2}\right] + \frac{w_\tau}{(1 - w_\tau)\tilde{P}^{N\tau} + w_\tau} \left[\frac{w_\tau}{2}\right]$$

The solution to (14) for w_τ is

$$(15) \qquad w_\tau = \frac{\sqrt{1/\tilde{P}^{N\tau}} - 1}{(1/\tilde{P}^{N\tau}) - 1} .$$

Eq. (15) gives the evolution of wages for individuals who do not receive any alternative offers through τ periods. Since w_τ equals the expected output of a worker who is not raided through τ rounds, it is also true that

$$(16) \qquad E(M | \text{unraided through } \tau \text{ rounds}) = \frac{\sqrt{1/\tilde{P}^{N\tau}} - 1}{(1/\tilde{P}^{N\tau}) - 1} .$$

Eq. (16) is easily interpreted: It is easily shown that

$$\lim_{P \to 0} \frac{\sqrt{1/\tilde{P}^{N\tau}} - 1}{(1/\tilde{P}^{N\tau}) - 1} = 1/2 .$$

If $P = 0$ (so that $\tilde{P} = 1$), the failure of a worker to be raided is uninformative. No outsider could have any information so remaining unraided carries no stigma. Thus, the expected level of M is 1/2, the same as the unconditional expectation of M for the entire population.

Conversely,

$$\lim_{P \to 1} \frac{\sqrt{1/\tilde{P}^{N\tau}} - 1}{(1/\tilde{P}^{N\tau}) - 1} = 0 .$$

If $P = 1$ (so that $\tilde{P} = 0$), then outsiders are always informed. Given any choice of w_τ by the current firm, all workers with $M > w_\tau$ are stolen away. This implies that the remaining workers have $M < w_\tau$. Under these circumstances the only w_τ that does not result in losses for the current firm is $w_\tau = 0$. Winner's curse operates at full force and the original firm takes that into account in setting the initial wage. All workers are raided and each worker receives a wage equal to his marginal product. As P goes to 1, the message and stigma associated with the failure to be raided grows stronger.

A number of implications follow immediately. First, and somewhat trivially, the relation between observed wage and actual marginal product becomes tighter, the higher is P. The more information that is had by outsiders, the more often the current firm is forced to pay a worker his marginal product. In the two extreme cases, when $P = 0$, all workers were paid $w_\tau = 1/2$ so the distribution of ability has infinite variance relative to the distribution of wages. When $P = 1$, each worker is paid M so the distribution of wages replicates the distribution of ability.

Second, the expected value of output for unraided workers is lower in later rounds than in early ones. Differentiating (16) with respect to τ, one obtains that[19]

$$\frac{\partial w_\tau}{\partial \tau} = \frac{\partial E(M|\text{ unraided through } \tau \text{ rounds})}{\partial \tau} < 0 .$$

The longer the duration during which the worker fails to receive an offer, the lower is his expected value. Recall that there are two reasons that a worker remains unraided: Rivals are uninformed or $M < w_\tau$. For any given w, the proportion of workers who are unraided because of rivals' ignorance falls with time since rivals are more likely to be informed as time progresses. Thus, the conditional expectation of unraided workers' output falls over time so w_τ must decline with τ.

This leads to an important implication: The quality of raided workers falls over time. Initially, only the most able workers get outside offers. Everyone seems to be going after the best workers. Uncertainty over ability, makes them the bargains in the labor market. As w_τ falls with τ, i.e., as the firm updates its evaluation of the worker's worth, successively lower ability workers become susceptible to raid. (Recently, firms who made job offers to Harvard MBAs tied the salary offer inversely to the length of time before acceptance. One interpretation is that the firms recognized that only lower quality workers remain in the job market as time progresses and lower the wage accordingly.)

The claim that the best workers are raided first is a statement that is made conditional upon observables. It does not imply, for example, that more highly educated workers are raided first. It implies only that among those with a given level of education, the best in the group are raided. This also implies that not conditioning upon anything, the higher ability workers are raided first across the entire population. The reason is that if ability is given by y, then the estimate of y based on observable x's must have the property that x's and the estimated residuals are uncorrelated. Since raids are based upon residuals (all observed information having already affected the initial wage offer), there is no correlation between raids and the observed x's. High residual workers are raided and they are, on average, of higher ability than the unraided ones.

This result is another manifestation of the Peter Principle. The best workers are stolen away or promoted out of the job. Those who remain are of lower average quality so they appear "incompetent" relative to the entire population of workers doing that job.

The way that stigma operates depends upon the probability of an outside offer. As already pointed out, if P is close to zero, then the failure to receive an outside offer carries little informational content so the wages of assembly line workers are likely to be less variable across workers than those of research academicians. Since researchers publish their thoughts, P, the probability that an outsider learns of a worker's productivity is quite high. But it is very unlikely that a star assembly line worker at GM is likely to be discovered by Ford. From (8), the expectation of M given no raid in round 1 is likely to be closer to the unconditional expectation for assembly line workers than for research economists. This is a step toward a theory of occupation wage differentials,[20] but more toward a theory of wage dispersion within an occupation.

Another wage differential is of interest. It is useful to examine how the difference between wages of raided workers and those of unraided workers varies with the likelihood of discovery, the number of potential employers, and experience (P, N and τ). To do this, it is necessary to derive the expected value for M for workers who <u>are</u> raided in round τ.

Workers who were raided in round τ and not before fall into one of two categories. Either they were susceptible to raid before period τ, but were undiscovered. This occurs with probability $\tilde{P}^{(\tau-1)N}(1 - w_{\tau-1})$. Or they have just become susceptible to raid in round τ because $w_{\tau-1} > M > w_\tau$. This occurs with probability $(w_{\tau-1} - w_\tau)$. Thus, the expected output of workers who are raided in round τ is

(17)

$$E_\tau \equiv E\left(M \middle| \begin{matrix}\text{raided in} \\ \text{round } \tau\end{matrix}\right) = \frac{\tilde{P}^{(\tau-1)N}(1-w_{\tau-1})[(1+w_{\tau-1})/2] + (w_{\tau-1} - w_\tau)\left[\frac{w_{\tau-1} + w_\tau}{2}\right]}{\tilde{P}^{(\tau-1)N}(1 - w_{\tau-1}) + (w_{\tau-1} - w_\tau)} \quad \text{for } \tau \geq 2$$

$$= (1 + w_1)/2 \qquad \text{for } \tau = 1$$

Raided workers receive their marginal products (at the latest by the following round) so their average wage is also given by the R.H.S. of (17).

It is difficult to characterize analytically the behavior of (17). However, a series of simulations is informative and illustrates the importance of stigma that results when one is unwanted by others. Table 2 presents the results.

The first panel of table 2 sets $P = .001$ and $N = 3$. This can be thought of as the extreme assembly-line-worker-in-a-big-three-auto-maker case. (Actually, the number of firms is $4 = N + 1$.) Note that even after 15 rounds, the conditional expectation and wage of M for unraided workers is only trivially different from the unconditional expectation of .5. The failure of an assembly-line worker to be raided by another firm is not informative because the probability that a high-quality worker will be discovered by a rival firm is small.

At the other extreme, in the last panel, is the case where $P = 1$. Here, number of firms is irrelevant because each outsider always has perfect information. There is no equilibrium wage offer greater than zero that does not result in losses to the current employer. Thus, $w(\tau) = 0$ and the entire population is raided.[21]

The research academician might fit somewhere in the middle. If the number of rival firms equals 10 and $P = .2$ (panel 6), then by the beginning of the third round, the expectation of unraided workers' output has fallen to the lowest 3.4% of the ability distribution. If this characterizes the true situation, it implies that outside offers are an extremely important signal. Those who do not receive them are treated very differently from those who do. E.g., the average wage of workers who are raided in round 1 is .62 as compared with .034 for those unraided at the end of three rounds. (Of course, the length of a round, although conceptually well defined, is difficult to calibrate empirically. A round is that period during which information remains private. Whether that is a month or a year is an open question.)

There is no monotonic relation that is independent of P between date of turnover and the wage change that occurs with the job switch. This can be seen by comparing the "Difference" columns of table 2 when $P = .001$, to that column when $P = .2$ for a given N.[22]

It is now possible to put everything together so as to derive turnover behavior. First, the total number of raids in any period τ consists of the sum of raids of all groups subject to raid. For example, in round 1, the only group that can be raided are those whose $M > w_1$. The expected number of raids on a worker in round 1 is

$$
\begin{aligned}
R(1) &= [\text{Prob}(M > w_1)][\text{Expected number of raids} \mid M > w_1] \\
&= (1 - w_1)NP \, .
\end{aligned}
$$

In round 2, two groups are subject to raids: those with $M > w_1$ and those with $w_2 < M < w_1$.

Eq. (7) already provides the general formula for $R(\tau)\big|_{M>w_1}$. It is tedious to derive a similar formula for the group with $E_t < M < E_{t-1}$ so it is derived in the appendix and presented in (18):

$$
\text{(18)} \qquad R(\tau)\Big|_{w_t < M < w_{t-1}}
$$

$$
\begin{aligned}
&= 0 && \text{for} \quad \tau < t \\
&= (1 - \tilde{P}^t)N && \text{for} \quad \tau = t \\
&= N\tilde{P}^{Nt}[P + (1-\tilde{P}^t)N(P + \theta(1-P))] && \text{for} \quad \tau = t+1 \\
&= NP\tilde{P}^{(\tau-1)N}[1 + N(P + \theta(1-P))] && \text{for} \quad \tau > t+1.
\end{aligned}
$$

Given (1) the total number of expected raids in any period τ is the probability weighted sum of raids on each of the groups. Thus:

$$
\text{(19)} \quad \begin{array}{l}\text{Total Expected Raids} \\ \text{on workers of unknown} \\ \text{quality in period } \tau\end{array} \equiv \overline{R(\tau)} = \sum_{t=1}^{\tau} (w_{t-1} - w_t)R(\tau)\Big|_{w_t < M < w_{t-1}} \, .
$$

where $w_0 \equiv 1$. The heaviest weight goes to the group with $M > w_1$. All other groups have decreasingly important probability weights in (19). This implies that the pattern observed for the labor market as a whole will closely resemble that of figure 1. But again, some simulations are instructive.

First, it is interesting to compare the pattern of raids that workers in different categories experience. Figure 2 shows the pattern of raids on workers with $w_t < M < w_{t-1}$ as t varies: the values of the parameters are $P = .1$, $N = 5$, $\theta = 1$.

There are two patterns that stand out. First and most obvious, is that for higher values of t, that is, for the group, say, with $w_5 < M < w_4$ as opposed to $w_1 < M < 1$ (panel 4 vs. panel 1), the raids start later. For the group with $w_5 < M < w_4$, raids do not begin until the fifth round whereas they begin in the first round for the group with $w_1 < M < 1$. Lower-quality workers tend to be raided later in their careers.

Second, the pattern changes as t increases. There is a tendency to get more raids in the initial raiding period because information accumulates before it becomes profitable to make a move. For example, consider an individual with $w_{10} < M < w_9$ (panel 5). It takes ten periods before the wage that unraided workers receive drops low enough to make a raid attempt successful. But during those ten periods, information accumulates on the worker so that when the tenth period arrives, many are able to make a move.

Of course, the effect of these spikes is likely to be extremely small in the aggregate for two reasons: First, the number of workers in any given cell, $w_t < M < w_{t-1}$ is very small (except for $t = 1$). Second, most workers are in situations where P, the probability of being discovered, is also likely to be low. So for example, in figure 2, panel 7, if $P = .001$, $N = 10$, $\theta = 1$ and $t = 10$, the size of the spike is small--only .1 raids occur in period 10. With such a low P, inference is more important than discovery and the period during which most raids occur is period 11. But even there less than one expected raid occurs. When we take into account that only $w_9 - w_{10}$ of the workers are in this category, the observed effect is trivial.

The entire picture is put together in figure 3. Figure 3 simulates equation (19) for different values of P, N, and θ. It represents the expected pattern of raids on a worker of unknown quality over time. As such, it is the sum of raids on each group $w_t < M < w_{t-1}$ as t varies weighted by the probability that the worker falls into that group. Since the probability that a worker has M such that $w_1 < M < 1$, is largest among groups, the pictures tend to be dominated by the pattern of raids on workers of that groups.

Different patterns of raids are possible. The inverted U-shape is most likely when θ is close to 1 (so that inference is important), when P is small (so discovery is unlikely), and when N is small (so that discovery is unlikely). The exponentially declining pattern is most likely to occur when θ is close to zero (cf. panel 8 to 7, 2 to 1, 4 to 3, and 6 to 5) so that inference is unimportant, when P is large (cf. panels 3 to 5) so that discovery is important, and when N is large (cf. panel 3 to 7 to 1) so that discovery is important.

This provides some simple implications. High turnover occupations are those most likely to have high values of N and P. Both imply a tendency toward exponentially declining raiding and turnover with time. In low turnover occupations with few firms and little visibility (low N, low P), the pattern of raiding and turnover is likely to be inverted U-shaped. (This assumes that variations in θ are small across occupations.)

Stated alternatively, the pattern of raids allows the empiricist to identify P, θ and the length of the round. Of course, assumptions about the shape of the distribution of M are crucial for identification.

The probability of turnover is similar, but not identical to the probability of a raid. A raid is successful if and only if the current employer is uninformed. The probability of turnover in round τ on a worker with $w_t < M < w_{t-1}$ is derived in the appendix and given by

$$\text{(20)} \quad \text{Prob. Turnover } (\tau)\Big|_{W_t < M < W_{t-1}}$$

$$= 0 \qquad \text{for} \quad \tau < t$$

$$= \tilde{P}^{t}(1 - \tilde{P}^{Nt}) \qquad \text{for} \quad \tau = t$$

$$= \begin{matrix} \tilde{P}^{N(\tau-1)}(1 - \tilde{P}^{N})\tilde{P}^{\tau} + \left[N\tilde{P}^{N(\tau-1)}(1 - \tilde{P}^{(\tau-1)})(1 - \theta)\right] \\ \cdot\, [1 - (1 - \theta)^{N} + (1 - \theta)^{N}(1 - P^{N})] \end{matrix} \qquad \text{for} \quad \tau \geq t+1.$$

Then the probability of turnover for any randomly drawn worker is derived from (20), weighted by the probability that $w_t < M < w_{t-1}$:

$$\text{(21)} \quad \overline{\text{Prob. Turnover}(\tau)} = \sum_{t=1}^{\tau} (w_{t-1} - w_t)\left(\text{Prob. Turnover}(\tau)\Big|_{w_t < M < w_{t-1}}\right) .$$

II. Extensions and Further Implications

Wages and Tenure:

The relation of wages to tenure is not straightforward. Since it is the best workers who are stolen away most rapidly, those young workers who remain in their jobs are likely to be of lower quality. So, tenure and wages can be negatively related. But in a sample of older workers, those who most recently changed jobs are likely to have been raided later in life. They are the lower quality workers so that tenure should be positively related to wages among older workers. Which effect dominates in a sample of the entire population is ambiguous, but one implication is clear: The relation of tenure to wages becomes more positive with age. I know of no evidence on this point.[23]

Stigma and Unemployment:

"Stigma" is often used to refer to workers who suffer spells of unemployment and find that subsequent demand for their services is adversely affected. Indeed, there is a significant literature that attempts to analyze these spells and to determine whether they are the result of inherent worker heterogeneity or of the signalling effect of unemployment.[24]

This paper focuses primarily on job changes without unemployment. However, if the current firm, j, is reinterpreted as the state of unemployment and w_j is defined as the reservation wage, then the model applies to unemployed workers as well. The interpretation of τ is the length (or number) of spells of unemployment. As the worker is "unraided" out of unemployment for a longer period of time, w_j falls and so does the average wage of raided workers. Thus, individuals who leave the state of unemployment quickly (are raided during initial rounds) have the highest expected wages (see table 2). Those who are unemployed for longer periods have lower expected wages because they are, on average, lower ability workers and their reservation wages are lower on average.

Lower values of P and N reduce the bite of stigma. Lower N is interpreted as fewer potential buyers. Individuals who are unemployed during a recession should not experience as large a wage depression following a given spell of unemployment as those who are unemployed during an expansion. This is illustrated in table 2 by comparing the path of wages for raided workers (those who eventually become employed in this context) when $P = .01$, $N = 3$ to that when $P = .01$, $N = 100$. The intuition is clear: When there are few buyers in the market, little can be inferred from the failure of a particular worker to be discovered by one.

Similarly, low values of P imply that unemployment should have small effects on subsequent wages. Here P can be thought of as the probability that a given buyer discovers a given worker. The value of P is low when workers are not making themselves known to potential firms. Consequently, the effect of being out of work, but in school or even on vacation, should be less detrimental to future wages than that of being unemployed for the same period of time while actively searching. Active searchers have higher P's so more can be inferred about those who do not find jobs. Table 2 illustrates this point as well. Compare the path of average wages of raided workers when $P = .001$, $N = 3$ to that when $P = .2$, $N = 3$. Wages fall much more quickly with unemployment in the latter case.

Shelf Life, Product Quality,
Product Prices, and Clearance Sales:

The same model can be used to analyze the dynamics of product prices as they relate to shelf life. Consider the problem facing the used-car salesman. On average, he probably has better information than his customers about the cars he sells. But on any given car, a buyer may actually know more about that one car than the seller ($P_j > P_k$, but $P_k > 0$). Those cars that are purchased most quickly tend to be the ones that are underpriced relative to their value. The ones that remain on the lot are those that are overpriced relative to their value.

In the same way that firms infer that their unraided workers are lower quality than the estimate at the beginning of a round, so the car dealer learns that the cars that remain on the lot are lower quality than estimated. At the beginning of each day, he sets the price of the car, $w(\tau)$, based upon his evaluation of the observables: $w(\tau)$ is the price below which he is willing to keep the car. But $w(\tau)$ falls with time on the lot. This is what clearance sales are about. Goods that remain on the shelf for a long period of time are reduced in price to reflect the lower quality that consumers attach to the less desirable color or style, which was not captured by initial prices.

Additionally, the comparative statics with respect to P, N and θ are appropriate in the product case as well. If P is high so that buyers are well informed, price falls more rapidly with shelf life. In the wholesale market where buyers are the retailers and are themselves well informed, prices on goods that remain on the warehouse shelf should fall quickly with time, relative to those that remain on the retailer's shelf. Additionally, when N is low, price remains more invariant to shelf time. For example, the price of unusual houses for which the number of buyers is few should not fall as steeply with time on the market as more homogeneous houses.

III. Summary and Conclusions

The focus of this paper is on turnover that occurs without unemployment. Job turnover and the evolution of wages can be related to the size of the market, the probability that a worker's ability level is discovered, and the speed with which information travels through the market. The dispersion of wages, slopes of age-earnings profiles, and the force of stigma are all affected by these variables. A large number of buyers and easily identifiable ability imply that stigma operates quickly and that the best workers are raided first so that workers who do not receive outside offers find themselves treated very differently from those who do.

The Peter Principle, that workers are promoted to their level of incompetence, is an implication. Since the best workers are stolen or promoted out of the job early, the older ones who remain and do not move further up the job ladder are low ability relative to the population of workers doing that job.

As a consequence of selective raiding, firms are left with lower than average quality workers. This insures that workers always choose to be employed by the best informed firm because that firm can offer the highest wage. This is true, independent of worker ability. Even low ability workers prefer to work at firms with the best information about worker ability. Thus, initial employers may appear to screen for the rest of the market.

Although unemployment is not the primary focus, a reinterpretation of the model provides implications for unemployed workers as well. Specifically, those who are unemployed during recession and those who do not actively seek jobs are likely to experience less adverse effects from the unemployment.

Finally, the model can be extended to examine product marketing practices. In particular, clearance sales and the general relationship between shelf life and product price can be understood. The prices of wholesale goods should fall more rapidly with time on the shelf than prices of retail goods. Similarly, the prices of heterogeneous goods in these markets should be more rigid with respect to time on the shelf.

APPENDIX

MATRIX OF GENERALIZED STRATEGIES AND RESULTS

The following matrix characterizes all possibilities when firms can be informed of the workers' ability, but also of other firms' beliefs. Define "cognizant" to mean that the firm knows whether or not the rival firm is informed. In each box is j or k, which tells where the worker ends up, and at what wage w. The number os a code for the discussion that follows.

k \ j		INFORMED		UNINFORMED	
		COG	NOT COG	COG	NOT COG
INFORMED	COG	1 j $w = M$	2 k $w = w_j + \varepsilon$	9 j $w = M$	10 k $w = w_j + \varepsilon$
INFORMED	NOT COG	3 j $w = w_j + \varepsilon$	4 k $w = w_j + \varepsilon$	11 j $w = w_j + \varepsilon$	12 k $w = w_j + \varepsilon$
UNINFORMED	COG	5 j $w = w_j$	6 j $w = w_j$	13 j $w = w_j$	14 j $w = w_j$
UNINFORMED	NOT COG	7 j $w = w_j$	8 j $w = w_j$	15 j $w = w_j$	16 j $w = w_j$

Discussion:

1: Both j and k know everything. j matches every offer from k so w is driven to M and the worker stays at j.

2: k knows that j is not cognizant. j can either try to deter any potential raids, but to do so requires $w_j = M$. It is better for j to stick to $w = w_j$ since $P_k < 1$ and $w_j = M$ results in zero quasi-rents. Thus, k picks off the worker at $w_j + \varepsilon$.

3: k is informed, but does not know that j is. Raids at $w_j + \varepsilon$. Since j is cognizant, j recognizes this as a true offer and matches, retaining the worker.

4: k raids at $w_j + \varepsilon$, but j does not know if threat is serious. Optimal response is to maintain $w = w_j$. Worker goes to k.

5. k is uninformed and realizes that j is informed. j maintains $w = w_j$ since this provides k with no information, retains worker. No raid by k at $w > w_j$ is profitable.

6: No raid by k at $w > w_j$ is profitable.

7: Same as 6.

8: Same as 6.

9: Although j is uninformed, he knows that k is informed. He infers M from k so worker stays at M at $w_j = M$. (No bluffing strategy is more profitable for k since M is the same at both firms.)

10: k is informed, but does not know that j is. So k's optimal strategy is to bid $w_j + \varepsilon$. Since j is cognizant, this is matched and worker stays at j.

11: j's optimal strategy is to ignore the worker's claim of an outside offer, so k gets worker at $w_j + \varepsilon$.

12: Same as 11.

13: Not profitable to raid at $w > w_j$ so worker stays at j.

14: Same as 13.

15: Same as 13.

16: Same as 13.

With the assumption that all informed firms are cognizant and no uninformed firms are cognizant, the matrix collapses to boxes 1, 7, 10, 16.

THE DERIVATION OF EQUATIONS

Equation 18

Consider the group with M such that $w_t < M < w_{t-1}$. No successful raid can occur in rounds 1 through $t - 1$ because an uninformed current employer pays $w_\tau > w_t > M$ for $\tau < t$. Thus,

$$R(\tau)\Big|_{w_t < M < w_{t-1}} = 0 \quad \text{for} \quad \tau < t\ .$$

During the $\tau < t$ rounds, however, firms are acquiring information on the worker. Thus in round t, the probability that a given firm will have become informed by then is $(1 - \tilde{P}^t)$. Since there are N firms, there are $N(1 - \tilde{P}^t)$ raids in round t via discovery. Thus,

$$R(\tau)\Big|_{w_t < M < w_{t-1}} = N(1 - \tilde{P}^t) \quad \text{for} \quad \tau = t\ .$$

Now, in period $\tau = t + 1$, either one raid occurred in period t, no raids occurred, or more than one raid occurred. If no raids occurred, then

$t + 1$ is like t. The probability of no raid in t is $\tilde{P}^{Nt}$. This branch yields NP raids in $t + 1$. The probability that there was exactly one raid in t is $N(1 - \tilde{P}^{t})\tilde{P}^{Nt}$ because there are N ways to have one firm informed and the N others uninformed. If that occurs, then θN firms learn by inference and $(1 - \theta)$ still may learn by discovery. So on this branch, there are

$$N[P + \theta(1 - P)]$$

expected raids. Thus,

$$R(\tau)\Big|_{w_t < M < w_{t-1}} = N\tilde{P}^{Nt}[P + (1 - \tilde{P}^{t})N(P + \theta(1 - P))] \quad \text{for} \quad \tau = t + 1.$$

For periods $\tau \geq t + 2$, a raid can only occur if there was no or one raid in $\tau - 1$. The probability that there is no raid in $\tau - 1$ is $\tilde{P}^{N(\tau-1)}$. This gives rise to NP expected raids in τ. If there was exactly one raid in $\tau - 1$, which happens with probability $NP\tilde{P}^{N(\tau-1)}$, then this gives rise to $\theta N + (1 - \theta)NP$ raids. Thus,

$$R(\tau)\Big|_{w_t < M < w_{t-1}} = NP\tilde{P}^{(\tau-1)N}[1 + N(P + \theta(1 - P))] \quad \text{for} \quad \tau \geq t + 2 .$$

So equation (18) is derived.

Equation 20

If $\tau > t$ such that $w_t < M < w_{t-1}$, then the probability of turnover is zero since no raids occur.

If $\tau = t$, then a turnover occurs iff the current employer is uninformed and at least one outsider is informed. This occurs with probability

$$\tilde{P}^{t}(1 - \tilde{P}^{Nt}) \ .$$

If $\tau \geq t + 1$, then turnover can occur because no turnover occurred through $\tau - 1$, at least one outsider becomes informed in τ and the current employer remains uninformed. This occurs with probability

$$\tilde{P}^{N(\tau-1)}(1 - \tilde{P}^{N})\tilde{P}^{\tau} \ .$$

Alternatively, exactly one raid could have occurred in $\tau - 1$ and that firm could have "forgotten" M which occurs with probability $(1 - \theta)$ while another firm learns it either by inference or discovery. That probability is

$$\underset{\substack{\text{exactly one raid}\\ \text{in } \tau - 1}}{N\tilde{P}^{N(\tau-1)}(1 - \tilde{P}^{(\tau-1)})}\underset{\substack{\text{raider}\\ \text{forgets}}}{(1 - \theta)}[\underset{\substack{\text{someone else}\\ \text{infers}}}{(1 - (1 - \theta)^{N})} + \underset{\substack{\text{no one infers but}\\ \text{someone else discovers}}}{(1 - \theta)^{N}(1 - \tilde{P}^{N})}]$$

Thus,

$$\text{(20)} \quad \text{Prob. Turnover}(\tau)\Big|_{W_t < M < w_{t-1}} = 0 \qquad \text{for } \tau < t$$

$$= \tilde{P}^{t}(1 - \tilde{P}^{Nt}) \qquad \text{for } \tau = t$$

$$= \tilde{P}^{N(\tau-1)}(1 - \tilde{P}^{N})\tilde{P}^{\tau} + \left[N\tilde{P}^{N(\tau-1)}(1 - \tilde{P}^{(\tau-1)})(1 - \theta)\right] \cdot [1 - (1 - \theta)^{N} + (1 - \theta)^{N}(1 - P^{N})] \qquad \text{for } \tau \geq t+1.$$

Footnotes

*Helpful comments were provided by Dennis Carlton, David Card, Richard Freeman, Merton Miller, Melvin Reder, John Riley, Sherwin Rosen, and Robert Topel. Work on this paper was supported in part by the National Science Foundation.

[1]See Phelps (1970), for a collection of these early papers.

[2]See Doeringer and Piore (1981), Thurow (1972),

[3]The most notable model of wage determination is Becker (1975). Others include Lazear (1979), Lazear and Rosen (1981), and Harris and Holmstrom (198-). The model that most effectively deals with job turnover in the absence of unemployment is Jovanovic (1979). Although the theory of specific human capital attempts to integrate wage dynamics with labor turnover, too much indeterminacy remains to have a very informative set of predictions. This is rectified somewhat by the work of Kuratani (1973), Hashimoto (1979), Hashimoto and Yu (1980) and Hall and Lazear (1982).

[4]See Peter (1969).

[5]All that this requires is a small moving cost.

[6]See, for example, Wilson (1977), Milgrom and Weber (1982), and Riley and Samuelson (1981) for a more complete discussion.

[7]This ignores any firm-specific aspects of the problem.

[8]See Harris and Townsend [1981] for a discussion of incentive compatibility and invertibility.

[9]This is a direct result of the discrete nature of information. In a more continuous setup, it is possible that $w_j(M)$ with $w_j' > 0$ might serve a deterrence role. But this complicates the problem greatly without any obvious changes in implications.

[10] A fixed w_j is also least susceptible to moral hazard problems where the firm tries to deceive the worker into believing that M is low. Also, it best masks j's knowledge from k.

[11] That $w_j > 0$ is obvious since $1 - P_k < 1$ so both $1/(1 - P_k)$ and $\sqrt{1/(1 - P_k)}$ exceed 1. It is also true that $w_j \leq \frac{1}{2}$. To see this, define $x \equiv 1/(1 - P_k)$ and assume the opposite: Then

$$(\sqrt{x} - 1)/(x - 1) > \frac{1}{2}$$

or

$$\sqrt{x} > \frac{1}{2}x + \frac{1}{2} \ .$$

This implies

$$0 > x^2 - 2x + 1 \ .$$

But $x^2 - 2x + 1$ has a min. at $x = 1$ where the value of $x^2 - 2x + 1 = 0$. Thus, a contradiction is obtained, so $w_j \leq \frac{1}{2}$.

[12] This ignores the kind of bargaining problem between worker and firm that Mortensen (1978) discusses. Rubinstein (1982) solves that problem when the value of the good is known to both parties, but the essence of the problem is that even if the worker knows M, and the seller knows that the worker knows M, there is uncertainty as to whether the firm knows M. Recall that the firm is only informed P_j of the time so $(1 - P_j)$ the time only knows the distribution and w_j is the optimum under these circumstances. This means that $1 - P_j$ of the time, a worker who demands $w > w_j$ will be let go. For most reasonable values of P_j (likely to be small), it is optimal for the worker merely to accept w_j. There are two caveats: First, if the worker costlessly and immediately can obtain another job that pays w_j, then all workers with $M > w_j$ try to bargain. Second, if the demand by the worker conveys the appropriate information to the firm about M, it may pay to

bargain even if the firm is uninformed (see the discussion by Fudenberg and Tirole (1983) on the effects of adding a period to a game). Note also that the higher is M, the more the worker has to gain so the more likely is the worker to demand $w > w_j$.

[13]See Grossman (1976), Grossman and Stiglitz (1976, 1980), Carlton (1982), and Gould and Verrecchia (1983) for examples of drawing inferences from observable market variables.

[14]The same kind of model can be used to explain "runs." One individual makes a discovery, and the rest imitate his behavior. Similarly, the ideas here are closely related to the literature on technological innovation and its diffusion across firms.

[15]The length of the round is defined to be that period over which information remains private.

[16]The same mechanism is at work in the market for firms, where takeover attempts tip off other potential buyers of the firm. As such, this framework can be used to explain takeover bid behavior as well (see Bradley, Desai, and Kim (1982), and Jarrell (1983)).

[17]This is similar to technology diffusion as analyzed by Spence (1982). Griliches (1982) discusses recent work on the issue. Telser (1982), who analyzes innovation, constructs an alternative model of technical change where diffusion of information plays a secondary role.

[18]Assume θ sufficiently large so that the probability that no firm learns by inference is close to zero.

[19]Proof that $\frac{\partial w_\tau}{\partial \tau} < 0$. Define $x(\tau) \equiv (1/\tilde{P}^{\tau N})$. Then

$$\frac{\partial w_\tau}{\partial \tau} = \frac{\partial w(\tau)}{\partial x(\tau)} \cdot \frac{\partial x(\tau)}{\partial \tau} .$$

Now,

$$\frac{\partial x(\tau)}{\partial \tau} = \tilde{P}^{-\tau N}(\ln \tilde{P})(-N) > 0$$

so the sign of $\frac{\partial w_\tau}{\partial \tau}$ is the same as that of $\frac{\partial x(\tau)}{\partial \tau}$. Also,

$$\frac{\partial w_\tau}{\partial x_\tau} = \frac{1 - \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{(x-1)^2} \leq 0 .$$

To see this, define $Y \equiv x^{1/2}$. Then the numerator is positive iff

$$2 > y + 1/y$$

which is a contradiction for any $y > 0$.

[20]See the pioneering work by Reder (1955) for an early attempt to explain differences in wages across occupations.

[21]This is the limiting case. If $P = 1$, then it is impossible for the current employer to be uninformed.

[22]There are a number of papers that examine empirically what happens to wages when a job change is made. The general finding seems to be that wage change on a job switch is greater for younger workers and often negative for older ones. See, for example, Borjas and Rosen (1980), Bartel (1980), Bartel and Borjas (1981).

[23]A complication is that tenure is also related to unemployment, with which the model only deals as an afterthought. For example, workers who suffer frequent spells of unemployment may have the lowest levels of tenure among all workers and they are likely to be the least able group.

[24]See, for example, Ellwood (1982); also Clark and Summers (1982) and Flinn and Heckman (1983).

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TABLE 2

Round	W(t)	Average Wage of Worker Raided in Round (τ)	Difference
P = .001, N = 3			
1	.4996	.7498	.2502
2	.4992	.7496	.2504
3	.4989	.7494	.2506
4	.4985	.7492	.2508
5	.4981	.7491	.2509
6	.4978	.7489	.2511
7	.4974	.7487	.2513
8	.4970	.7485	.2515
9	.4966	.7483	.2517
10	.4962	.7481	.2519
11	.4959	.7479	.2521
12	.4955	.7477	.2522
13	.4951	.7476	.2524
14	.4947	.7474	.2526
15	.4944	.7472	.2528
P = .01, N = 3			
1	.4962	.7481	.2519
2	.4925	.7462	.2537
3	.4887	.7442	.2555
4	.4849	.7423	.2574
5	.4812	.7403	.2592
6	.4774	.7384	.2610
7	.4736	.7365	.2628
8	.4699	.7345	.2646
9	.4661	.7326	.2664
10	.4624	.7306	.2682
11	.4586	.7287	.2700
12	.4549	.7267	.2718
13	.4512	.7248	.2736
14	.4474	.7228	.2754
15	.4437	.7209	.2772
P = .2, N = 3			
1	.4171	.7085	.2915
2	.3386	.6397	.3011
3	.2681	.5635	.2954
4	.2077	.4832	.2755
5	.1579	.4028	.2449
6	.1183	.3268	.2084
7	.0876	.2585	.1709
8	.0643	.2000	.1357
9	.0469	.1519	.1051
10	.0340	.1137	.0798
11	.0246	.0842	.0596
12	.0177	.0617	.0440
13	.0127	.0450	.0322
14	.0091	.0326	.0235
15	.0066	.0235	.0170

Round	W(t)	Average Wage of Worker Raided in Round (τ)	Difference
P = .001, N = 100			
1	.4875	.7437	.2563
2	.4750	.7369	.2619
3	.4625	.7299	.2674
4	.4501	.7230	.2728
5	.4378	.7160	.2782
6	.4255	.7089	.2834
7	.4133	.7018	.2885
8	.4013	.6947	.2935
9	.3893	.6876	.2983
10	.3775	.6805	.3030
11	.3658	.6733	.3075
12	.3543	.6662	.3119
13	.3429	.6590	.3161
14	.3317	.6518	.3201
15	.3207	.6446	.3239
P = .01, N = 100			
1	.3769	.6885	.3115
2	.2680	.5701	.3022
3	.1813	.4420	.2607
4	.1182	.3211	.2029
5	.0750	.2207	.1457
6	.0467	.1453	.0985
7	.0288	.0928	.0640
8	.0176	.0581	.0405
9	.0107	.0359	.0252
10	.0065	.0220	.0155
11	.0040	.0134	.0095
12	.0024	.0082	.0058
13	.0015	.0050	.0035
14	.0009	.0030	.0021
15	.0005	.0018	.0013
P = 2, $N \geq 1$			
1	0	.5	.5
2	0	.5	.5
3	0	.5	.5
4	0	.5	.5
5	0	.5	.5
6	0	.5	.5
7	0	.5	.5
8	0	.5	.5
9	0	.5	.5
10	0	.5	.5
11	0	.5	.5
12	0	.5	.5
13	0	.5	.5
14	0	.5	.5
15	0	.5	.5

Round	W(t)	Average Wage of Worker Raided in Round (τ)	Difference
P = .001, N = 10			
1	.4987	.7494	.2506
2	.4975	.7487	.2512
3	.4962	.7481	.2519
4	.4950	.7475	.2525
5	.4937	.7468	.2531
6	.4925	.7462	.2537
7	.4912	.7456	.2543
8	.4900	.7450	.2550
9	.4887	.7443	.2556
10	.4875	.7437	.2562
11	.4862	.7431	.2568
12	.4850	.7424	.2574
13	.4837	.7418	.2580
14	.4825	.7412	.2587
15	.4812	.7405	.2593
P = .01, N = 10			
1	.4874	.7437	.2563
2	.4749	.7368	.2619
3	.4624	.7298	.2675
4	.4499	.7228	.2729
5	.4375	.7158	.2783
6	.4252	.7087	.2835
7	.4130	.7016	.2886
8	.4008	.6945	.2936
9	.3888	.6873	.2985
10	.3769	.6801	.3032
11	.3652	.6730	.3077
12	.3537	.6657	.3121
13	.3423	.6585	.3163
14	.3310	.6513	.3203
15	.3200	.6411	.3241
P = .2, N = 10			
1	.2468	.6234	.3766
2	.0970	.3302	.2332
3	.0340	.1340	.1000
4	.0114	.0475	.0361
5	.0038	.0160	.0122
6	.0012	.0053	.0041
7	.0004	.0017	.0013
8	.0001	.0006	.0004
9	.0000	.0002	.0001
10	.0000	.0001	.0000
11	.0000	.0000	.0000
12	.0000	.0000	.0000
13	.0000	.0000	.0000
14	.0000	.0000	.0000
15	.0000	.0000	.0000

Figure 1

Panel 1

```
P= 1   N= 5   t= 1   theta= 1
round= 1  raids= 5.000    ***************************************************
round= 2  raids= 0.000
round= 3  raids= 0.000
round= 4  raids= 0.000
round= 5  raids= 0.000
round= 6  raids= 0.000
round= 7  raids= 0.000
round= 8  raids= 0.000
round= 9  raids= 0.000
round=10  raids= 0.000
round=11  raids= 0.000
round=12  raids= 0.000
round=13  raids= 0.000
round=14  raids= 0.000
round=15  raids= 0.000
```

Panel 2

```
P= .001   N= 3   t= 1   theta= 1
round= 1  raids= 0.003    *
round= 2  raids= 0.012    *
round= 3  raids= 0.012    *
round= 4  raids= 0.012    *
round= 5  raids= 0.012    *
round= 6  raids= 0.012    *
round= 7  raids= 0.012    *
round= 8  raids= 0.012    *
round= 9  raids= 0.012    *
round=10  raids= 0.012    *
round=11  raids= 0.012    *
round=12  raids= 0.012    *
round=13  raids= 0.012    *
round=14  raids= 0.012    *
round=15  raids= 0.012    *
```

Panel 3

```
P= .1   N= 3   t= 1   theta= 1
round= 1  raids= 0.300    ******
round= 2  raids= 0.875    ***************
round= 3  raids= 0.638    ***********
round= 4  raids= 0.465    ********
round= 5  raids= 0.339    ******
round= 6  raids= 0.247    *****
round= 7  raids= 0.180    ****
round= 8  raids= 0.131    ***
round= 9  raids= 0.096    **
round=10  raids= 0.070    **
round=11  raids= 0.051    *
round=12  raids= 0.037    *
round=13  raids= 0.027    *
round=14  raids= 0.020    *
round=15  raids= 0.014    *
```

Panel 4

```
P= .5   N= 3   t= 1   theta= 1
round= 1  raids= 1.500    **************************
round= 2  raids= 0.750    *************
round= 3  raids= 0.034    **
round= 4  raids= 0.012    *
round= 5  raids= 0.001    *
round= 6  raids= 0.000    *
round= 7  raids= 0.000    *
round= 8  raids= 0.000    *
round= 9  raids= 0.000    *
round=10  raids= 0.000    *
round=11  raids= 0.000    *
round=12  raids= 0.000    *
round=13  raids= 0.000    *
round=14  raids= 0.000    *
round=15  raids= 0.000    *
```

Panel 5

```
P= .1   N= 20   t= 1   theta= 1
round= 1  raids= 2.000    ******
round= 2  raids= 5.106    *************
round= 3  raids= 0.621    **
round= 4  raids= 0.075    *
round= 5  raids= 0.009    *
round= 6  raids= 0.001    *
round= 7  raids= 0.000    *
round= 8  raids= 0.000    *
round= 9  raids= 0.000    *
round=10  raids= 0.000    *
round=11  raids= 0.000    *
round=12  raids= 0.000    *
round=13  raids= 0.000    *
round=14  raids= 0.000    *
round=15  raids= 0.000    *
```

Panel 6

```
P= .1   N= 20   t= 1   theta= .1
round= 1  raids= 2.000    ******
round= 2  raids= 1.167    ***
round= 3  raids= 0.142    *
round= 4  raids= 0.017    *
round= 5  raids= 0.002    *
round= 6  raids= 0.000    *
round= 7  raids= 0.000    *
round= 8  raids= 0.000    *
round= 9  raids= 0.000    *
round=10  raids= 0.000    *
round=11  raids= 0.000    *
round=12  raids= 0.000    *
round=13  raids= 0.000    *
round=14  raids= 0.000    *
round=15  raids= 0.000    *
```

Figure 2

Panel 1

```
p= .1   N= 5   t= 1   theta= 1
round= 1  raids= 0.500     ******
round= 2  raids= 1.771     *********************
round= 3  raids= 1.046     *************
round= 4  raids= 0.618     *******
round= 5  raids= 0.365     ****
round= 6  raids= 0.215     ***
round= 7  raids= 0.127     **
round= 8  raids= 0.075     *
round= 9  raids= 0.044     *
round=10  raids= 0.026     *
round=11  raids= 0.015     *
round=12  raids= 0.009     *
round=13  raids= 0.005     *
round=14  raids= 0.003     *
round=15  raids= 0.002     *
```

Panel 2

```
p= .1   N= 5   t= 2   theta= 1
round= 1  raids= 0.000
round= 2  raids= 0.950     ***********
round= 3  raids= 1.831     **********************
round= 4  raids= 0.618     *******
round= 5  raids= 0.365     ****
round= 6  raids= 0.215     ***
round= 7  raids= 0.127     **
round= 8  raids= 0.075     *
round= 9  raids= 0.044     *
round=10  raids= 0.026     *
round=11  raids= 0.015     *
round=12  raids= 0.009     *
round=13  raids= 0.005     *
round=14  raids= 0.003     *
round=15  raids= 0.002     *
```

Panel 3

```
p= .1   N= 5   t= 3   theta= 1
round= 1  raids= 0.000
round= 2  raids= 0.000
round= 3  raids= 1.355     ****************
round= 4  raids= 1.498     ******************
round= 5  raids= 0.365     ****
round= 6  raids= 0.215     ***
round= 7  raids= 0.127     **
round= 8  raids= 0.075     *
round= 9  raids= 0.044     *
round=10  raids= 0.026     *
round=11  raids= 0.015     *
round=12  raids= 0.009     *
round=13  raids= 0.005     *
round=14  raids= 0.003     *
round=15  raids= 0.002     *
```

Panel 4

```
p= .1   N= 5   t= 5   theta= 1
round= 1  raids= 0.000
round= 2  raids= 0.000
round= 3  raids= 0.000
round= 4  raids= 0.000
round= 5  raids= 2.048     *************************
round= 6  raids= 0.771     *********
round= 7  raids= 0.127     **
round= 8  raids= 0.075     *
round= 9  raids= 0.044     *
round=10  raids= 0.026     *
round=11  raids= 0.015     *
round=12  raids= 0.009     *
round=13  raids= 0.005     *
round=14  raids= 0.003     *
round=15  raids= 0.002     *
```

Panel 5

```
p= .1   N= 5   t= 10   theta= 1
round= 1  raids= 0.000
round= 2  raids= 0.000
round= 3  raids= 0.000
round= 4  raids= 0.000
round= 5  raids= 0.000
round= 6  raids= 0.000
round= 7  raids= 0.000
round= 8  raids= 0.000
round= 9  raids= 0.000
round=10  raids= 3.257     ***************************************
round=11  raids= 0.086     *
round=12  raids= 0.009     *
round=13  raids= 0.005     *
round=14  raids= 0.003     *
round=15  raids= 0.002     *
```

Panel 6

```
p= .1   N= 5   t= 14   theta= 1
round= 1  raids= 0.000
round= 2  raids= 0.000
round= 3  raids= 0.000
round= 4  raids= 0.000
round= 5  raids= 0.000
round= 6  raids= 0.000
round= 7  raids= 0.000
round= 8  raids= 0.000
round= 9  raids= 0.000
round=10  raids= 0.000
round=11  raids= 0.000
round=12  raids= 0.000
round=13  raids= 0.000
round=14  raids= 3.856     **********************************************
round=15  raids= 0.012     *
```

Panel 7

```
p= .001   N= 10   t= 10   theta= 1
round= 1  raids= 0.000
round= 2  raids= 0.000
round= 3  raids= 0.000
round= 4  raids= 0.000
round= 5  raids= 0.000
round= 6  raids= 0.000
round= 7  raids= 0.000
round= 8  raids= 0.000
round= 9  raids= 0.000
round=10  raids= 0.100     *
round=11  raids= 0.910     *****
round=12  raids= 0.099     *
round=13  raids= 0.098     *
round=14  raids= 0.097     *
round=15  raids= 0.096     *
```

Figure 3

Panel 1

```
p= .1    N= 10    theta= 1
round= 1   raids= 0.629     ****
round= 2   raids= 2.626     **************
round= 3   raids= 1.353     *******
round= 4   raids= 0.665     ****
round= 5   raids= 0.337     **
round= 6   raids= 0.183     *
round= 7   raids= 0.105     *
round= 8   raids= 0.063     *
round= 9   raids= 0.039     *
round=10   raids= 0.024     *
round=11   raids= 0.015     *
round=12   raids= 0.009     *
round=13   raids= 0.006     *
round=14   raids= 0.003     *
round=15   raids= 0.002     *
```

Panel 2

```
p= .1    N= 10    theta= .1
round= 1   raids= 0.629     ****
round= 2   raids= 0.850     *****
round= 3   raids= 0.523     ***
round= 4   raids= 0.328     **
round= 5   raids= 0.212     **
round= 6   raids= 0.138     *
round= 7   raids= 0.090     *
round= 8   raids= 0.058     *
round= 9   raids= 0.037     *
round=10   raids= 0.023     *
round=11   raids= 0.014     *
round=12   raids= 0.009     *
round=13   raids= 0.005     *
round=14   raids= 0.003     *
round=15   raids= 0.002     *
```

Panel 3

```
p= .1    N= 100    theta= 1
round= 1   raids= 9.949     *****
round= 2   raids= 0.124     *
round= 3   raids= 0.001     *
round= 4   raids= 0.000     *
round= 5   raids= 0.000     *
round= 6   raids= 0.000     *
round= 7   raids= 0.000     *
round= 8   raids= 0.000     *
round= 9   raids= 0.000     *
round=10   raids= 0.000
round=11   raids= 0.000
round=12   raids= 0.000
round=13   raids= 0.000
round=14   raids= 0.000
round=15   raids= 0.000
```

Panel 4

```
p= .1    N= 100    theta= .1
round= 1   raids= 9.949     *****
round= 2   raids= 0.102     *
round= 3   raids= 0.001     *
round= 4   raids= 0.000     *
round= 5   raids= 0.000     *
round= 6   raids= 0.000     *
round= 7   raids= 0.000     *
round= 8   raids= 0.000     *
round= 9   raids= 0.000     *
round=10   raids= 0.000
round=11   raids= 0.000
round=12   raids= 0.000
round=13   raids= 0.000
round=14   raids= 0.000
round=15   raids= 0.000
```

Panel 5

```
p= .01    N= 100    theta= 1
round= 1   raids= 0.623     *
round= 2   raids=23.251     ************
round= 3   raids=11.609     ******
round= 4   raids= 5.141     ***
round= 5   raids= 2.144     **
round= 6   raids= 0.890     *
round= 7   raids= 0.386     *
round= 8   raids= 0.182     *
round= 9   raids= 0.094     *
round=10   raids= 0.053     *
round=11   raids= 0.031     *
round=12   raids= 0.019     *
round=13   raids= 0.012     *
round=14   raids= 0.008     *
round=15   raids= 0.005     *
```

Panel 6

```
p= .01    N= 100    theta= .01
round= 1   raids= 0.623     *
round= 2   raids= 0.899     *
round= 3   raids= 0.579     *
round= 4   raids= 0.386     *
round= 5   raids= 0.266     *
round= 6   raids= 0.186     *
round= 7   raids= 0.129     *
round= 8   raids= 0.089     *
round= 9   raids= 0.061     *
round=10   raids= 0.041     *
round=11   raids= 0.027     *
round=12   raids= 0.018     *
round=13   raids= 0.012     *
round=14   raids= 0.008     *
round=15   raids= 0.005     *
```

Panel 7

```
p= .1    N= 30    theta= 1
round= 1   raids= 2.488     *****
round= 2   raids= 4.011     *******
round= 3   raids= 0.440     *
round= 4   raids= 0.078     *
round= 5   raids= 0.018     *
round= 6   raids= 0.004     *
round= 7   raids= 0.001     *
round= 8   raids= 0.000     *
round= 9   raids= 0.000     *
round=10   raids= 0.000     *
round=11   raids= 0.000     *
round=12   raids= 0.000     *
round=13   raids= 0.000     *
round=14   raids= 0.000     *
round=15   raids= 0.000     *
```

Panel 8

```
p= .1    N= 30    theta= .1
round= 1   raids= 2.488     *****
round= 2   raids= 1.448     ***
round= 3   raids= 0.299     *
round= 4   raids= 0.072     *
round= 5   raids= 0.018     *
round= 6   raids= 0.004     *
round= 7   raids= 0.001     *
round= 8   raids= 0.000     *
round= 9   raids= 0.000     *
round=10   raids= 0.000     *
round=11   raids= 0.000     *
round=12   raids= 0.000     *
round=13   raids= 0.000     *
round=14   raids= 0.000     *
round=15   raids= 0.000     *
```