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ORIGINAL RESEARCH PAPER

Statistics

COMPARISON OF INVENTORY MODELS UNDER NON-INSTANTANEOUS DETERIORATION RATE WITH PROBABILISTIC DEMAND AND SHORTAGES

KEY WORDS: Inventory, Non-instantaneous deterioration, Probabilistic Demand, Random Variable, Shortages.

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ABSTRACT

In this research, present an inventory model with non-instantaneous deterioration rate is consider i.e. depend on time. The purpose of this paper is to deliver a comparative study of diverse probabilistic environment, where demand is considered as a random variable follows probabilistic distributions. Shortages are allowed. The anticipated model is conveyed to highest profit of the formation. Mathematical analysis is carried out at the end to validate models. A parametric analysis is accomplished to explore the impact of key parameters.

INTRODUCTION

As we assume that demand is undefined, it is interesting to explore how demand uncertainty affect performance of the system. A several researchers have deliberated inventory models assuming the demand to be persistent, time reliant, depending on price, stock dependent and exponential etc. under with and without shortages under different environment like delay in payment, with effect of inflation and taking variable holding cost. When we consider uncertain demand then our focus is on probabilistic demand where demand is considered as a random variable follows probabilistic distributions. But very effective part of the inventory system is deterioration which plays very crucial role. Many of the researchers studied inventory model with fixed deterioration and different deterioration like part wise, depend on time. The study of inventory models for declining items was first christened by Whitin[18]. A model with relentless rate of deterioration was originally established by Ghare as well as Schrader [7]. Raffat[14] categorized deterioration by time value of inventory. In the last few decades many researchers have done a lot of research work in studying and analyzing the different kinds of deteriorating inventory systems under different situations. From past few years the analysis of deteriorating inventory problems has expected a abundant deal of attention. Nahmias[12] divided the deteriorating inventory problems into given groups like fixed lifespan and random lifespan. Harris [10] first developed the EOQ formula with the help of differential calculus and same formula was again derived by Wilson[17]. Lots of works done in effect of deterioration. Covert and Philip [4] prolonged Schrader and Ghare continual deterioration rate to a two-parameter Weibull distribution. After years, there are quite a few remarkable papers allied to deterioration with and without shortages like Jaiswal and Shah[15], Giri along with Goyal [8], Bhunia with Maiti [2], Chung with Tsai[3] and Patelat.el. [13]. Sheikh and Patel [16] develop inventory model with altered deterioration rates. Aggarwal [1] deliberated an order level inventory model through continual rate of worsening. So many researcher are focus on probabilistic inventory models. Some of them have done their work with demand follows probabilistic distribution. Gupt[9] develop production model with probabilistic demand. Comparative study under different environment of inventory model was studied by Durai and Chakrabarti [5]. Multi-item inventory model with probabilistic demand derived by Kar, Roy and Maiti[11]. Fergany[6] studied probabilistic multi-item inventory form along with scarcities under limitations.

Herein paper we pay attention to an inventory model with probabilistic demand follows Uniform, Triangular and exponential distribution with their mean. Non-instantaneous deterioration rate to be taken i.e. during the system deterioration depends on time. The model is represented with numerical example and a parametric analysis with tolerance limits. The goal of given analysis is to create relative inventory model that the firm gets enhanced profit comparative to profitable surroundings. An approach is accomplished, in which the entire profit in that path is maximized. Hence the optimal era of evaluation as well total optimal aim of inventory stage is resulted.

NOTATIONS

D(t):Demand x is a random variable follows probability distribution(Mean demand as .)

A: Ordering cost

C_h: Holding cost per unit time

C_s: scarcity cost per unit

C_c: Purchasing cost per unit

C_p: Promotion price per unit

T: Total cycle of in time

θt: Deterioration rate during $0 \le t \le t_1$

I(t): Inventory level at time t

Q_1: Inventory level initially

Q_2: Inventory level at T

Π: Profit

ETP: Expected total profit

ASSUMPTIONS

(i) The demand rate of the goods is taken as probabilistic.

(ii) Non-instantaneous deterioration rate is taken i.e. θt dependent on time.

(iii) Shortages are allowed and totally backlogged.

(iv) Single item inventory is considered.

(v) Restoration or replacement of declining items are excluded.

(vi) Holding cost is stable.

DEVELOPMENT AND ANALYSIS OF THE MODEL

Below diagram shows the nature of inventory at time 0 to T. The inventory level gradually diminishes due to demand and deterioration during the phase $[0,t_1]$ and ultimately falls to zero at $t = t_1$. During the phase $[t_1, T]$ shortages occur which is completely backlogged.

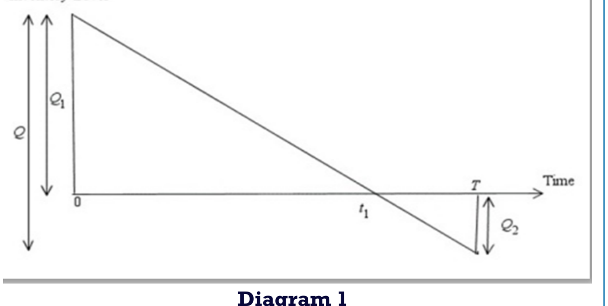


Diagram 1

The inventory sets are describe by the differential equations:

$$\frac{dI'(t)}{dt} + \theta t I'(t) = -\mu(x), \quad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI''(t)}{dt} = -\mu(x), \quad t_1 \le t \le T \qquad (2)$$

With initial orders $I'(0)=Q_1, I'(t_1)=0$ and $I''(T)=-Q_2$.

Solving above equation (1) and (2) we have,

$$I'(t)=\mu(x)\left(-(t-t_1)+\frac{1}{6}\theta\left(t_1^3-t^3\right)\right)\left(1-\frac{1}{2}\theta t^2\right),\quad 0\le t\le t_1 \quad (3)$$

$$I''(t)=\mu(x)\left(-(t-t_1)\right),\quad t_1\le t\le T \quad (4)$$

Substitute $t=0$ in equation (3) and $t=T$ in equation (4) we get quantity as,

$$Q_1=\mu(x)\left(t_1+\frac{1}{6}\theta t_1^3\right) \quad (5)$$

$$-Q_2=\mu(x)(T-t_1), \quad (6)$$

The following cost characterize the whole inventory structure.

$$(c_1)\ \text{Ordering Cost}=A \quad (7)$$

$$(c_2)\ \text{Holding Cost}=C_h\int_0^{t_1} I(t)dt \quad (8)$$

$$(c_3)\ \text{Deterioration Cost}=C_c\int_0^{t_1}\theta\, t\, I(t)dt \quad (9)$$

$$(c_4)\ \text{Shortage Cost}=-C_s\int_{t_1}^{T} I(t)dt \quad (10)$$

$$(c_5)\ \text{Sales Revenue}=C_p\int_0^{T}\mu(x)dt \quad (11)$$

The total profit function can be written as for inventory system as,

$$\Pi(t_1,T)=\frac{c_5-\sum_{i=1}^{4}c_i}{T}$$

$$\Pi(t_1,T)=\frac{1}{T}\left[\begin{array}{l} pT\mu(x)-A+C_s\begin{pmatrix}-\frac{1}{2}\mu(x)\left(T^2-t_1^2\right)\\ +\mu(x)t_1(T-t_1)\end{pmatrix}\\ -C_h\begin{pmatrix}\frac{1}{72}\mu(x)\theta^2t_1^6+\frac{1}{12}\mu(x)\theta t_1^4\\ -\frac{1}{6}\mu(x)\left(t_1+\frac{1}{6}\theta t_1^3\right)\theta t_1^3\\ -\frac{1}{2}\mu(x)t_1^2+\mu(x)\left(t_1+\frac{1}{6}\theta t_1^3\right)t_1\end{pmatrix}\\ -C_c\begin{pmatrix}\frac{1}{84}\mu(x)\theta^3t_1^7+\frac{1}{15}\mu(x)\theta^2t_1^5\\ -\frac{1}{8}\mu(x)\left(t_1+\frac{1}{6}\theta t_1^3\right)\theta^2t_1^4\\ -\frac{1}{3}\mu(x)\theta t_1^3+\frac{1}{2}D(t)\left(t_1+\frac{1}{6}\theta t_1^3\right)\theta t_1^2\end{pmatrix}\end{array}\right] \quad (12)$$

Substitute the values of cost stated in equation (7) to (11), we get total profit per unit with t1 and T. Differentiate equation (12) by virtue of t1 and T, compare it to zero, i.e.

$$\frac{\partial\Pi}{\partial t_1}=0,\ \frac{\partial\Pi}{\partial T}=0 \quad (14)$$

Provided it fulfills the second order rule that

$$\begin{vmatrix}\frac{\partial^2\Pi}{\partial^2 t_1} & \frac{\partial^2\Pi}{\partial t_1\partial T}\\ \frac{\partial^2\Pi}{\partial t_1\partial T} & \frac{\partial^2\Pi}{\partial^2 T}\end{vmatrix}>0 \quad (14)$$

Following are the cases for different demand function,

MODEL I

X~Uniform Distribution $f(x)=\frac{1}{D_b-D_a}, D_a\le X\le D_b$

then the mean demand $\mu(x)=\frac{D_a+D_b}{2}$,

then Expected Total Profit is:

$$ETP(t_1,T)==\left[\frac{c_5-\sum_{i=1}^{4}c_i}{T}\right]_{\mu(x)=\frac{D_a+D_b}{2}} \quad (15)$$

MODEL II

X~Triangular Distribution

$$f(x)=\begin{cases}0, & \text{for } x<D_a\\ \frac{(x-D_a)2}{(D_b-D_a)(D_c-D_a)}, & \text{for } D_a\le x\le D_c\\ \frac{2}{D_b-D_a}, & \text{for } x=D_c\\ \frac{(D_b-x)2}{(D_b-D_a)(D_b-D_c)}, & \text{for } D_c<x\le D_b\\ 0, & \text{for } D_b<x\end{cases}$$

then mean demand is $\mu(x)=\frac{1}{3}(D_a+D_b+D_c)$,

Hence Expected Total Profit is:

$$ETP(t_1,T)=\left[\frac{c_5-\sum_{i=1}^{4}c_i}{T}\right]_{\mu(x)=\frac{1}{3}(D_a+D_b+D_c)} \quad (16)$$

MODEL III

$X\sim$ Exponential Distribution $f(x)=De^{-Dx},\ 0\le x\le\infty$

then mean demand as $\mu(x)=\frac{1}{D}$,

then Expected Total Profit is:

$$ETP(t_1,T)=\left[\frac{c_5-\sum_{i=1}^{4}c_i}{T}\right]_{\mu(x)=\frac{1}{D},} \quad (17)$$

NUMERICAL ANALYSIS

Taking appropriate values for various input parameters for each model, we determine optimal time and profit.

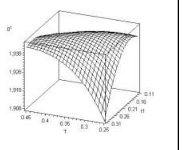
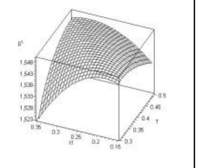
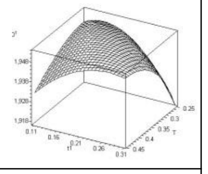
Considering $A=Rs.100, C_h=Rs.5, C_s=Rs.8, C_c=Rs.25, C_p=Rs.40, \theta=0.05$

Table (I)

	Model		
	I	II	III

	Uniform $D_a = 400$ $D_b = 600$	Triangular $D_a = 300$ $D_b = 400$ $D_c = 500$	Exponential D = 0.001
t_1	0.2171	0.2421	0.2165
T	0.3565	0.3981	0.3556
Profit	19442.16	15500.72	19536.35
Q_1	108.59	96.88	108.80
Q_2	69.70	62.39	69.88

The second order regulation agreed in equation (14) is gratified. The three dimensional demonstration of concavity on expected total profit are correspondingly display for all the model.

Model I	Model II	Model III
Uniform Demand	Triangular Demand	Exponential Demand
		
Graph (i)	Graph (ii)	Graph (iii)

PARAMETRIC ANALYSIS

The parametric analysis of the key parameter has been discussed below:

Table (ii) Parametric Analysis

Parameter change (%)		t_1	T	Profit
Uniform Distribution				
D_a and D_b	-50	0.3042	0.5019	9604.63
	-25	0.2498	04109	14516.47
	+25	0.1946	0.3192	24376.68
	+50	0.1779	0.2917	29317.49
Triangular Distribution				
D_a, D_b and D_c	-50	0.3389	0.5601	7646.04
	-25	0.2784	0.4588	11567.19
	+25	0.2171	0.3565	19442.16
	+50	0.1985	0.3258	23389.22
Exponential Distribution				
D	-50	0.1097	0.1792	78887.45
	-25	0.1341	0.2193	52424.36
	+25	0.1724	0.2825	31295.21
	+50	0.1885	0.3092	26023.01

From the above parametric analysis, the following consideration can be made. For all distribution when we change in parameter decrease to increase the profit will increase. The profit should be maximum in Uniform and Exponential distribution compare to triangular distribution. We show there is increase-decrease in t_1 and T in all the three models.

CONCLUDING REMARKS

In this study, we represent the process to obtain the maximum profit of the entire system. We find profit for all the three models which have probabilistic demand and varying deterioration depends on time. We observe that there is maximum profit in model III where demand follows exponential distribution. Compare to exponential and uniform distribution there is less profit in triangular distribution. There are more than a few extension of this research work . The study may be extended to delay in payment, effect of inflation and multi-item EOQ model.

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