

# STUDY OF THE SENSITIVITY OF THE OFDM WIRELESS SYSTEMS TO THE CARRIER FREQUENCY OFFSET (CFO)

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## ABSTRACT

*As it has already been revealed, the OFDM systems are very sensitive to the CFO. CFO is one of the most important drawbacks of the OFDM wireless systems. CFO destroys the orthogonality relationship between the subcarrier and creates ICI in OFDM systems. ICI can create frequency mismatched between the transmitter and receiver; this frequency mismatch can lead to the severe performance degradation issue for OFDM wireless systems. However for having a reliable receiver the effect of the CFO must be estimated and compensated in receiver side. This paper reviews the reasons of the creating CFO and analyses its effects on the performance of the OFDM systems. The results clearly illustrate the effects of CFO.*

## KEYWORDS

*Carrier frequency offset (CFO), CFO estimation, OFDM performance, Frequency Offset, Inter-Carrier Interference (ICI)*

## 1. INTRODUCTION

About 55 years ago, Doelz et al. [1] published the idea of dividing the transmitting data into the number of interleaved bit streams and modulate numerous carriers, and then Chang [2] was the one that in the middle of the 60's, presented the basic idea of multicarrier modulation. But, it was Weinstein and Ebert [3] that brought it to the fruition; they showed how to apply the Discrete Fourier Transform (DFT) for performing the baseband modulation and demodulation. They showed how by using the DFT, we can increase and improve the efficiency of the modulation and demodulation. As we know, in order to efficiently modulate and demodulate the OFDM signals, a few pre and post tasks must be done which the most important of them are: Frequency synchronization, Carrier Frequency Offset (CFO), Peak-to-Average Power Reduction (PAPR), phase noise compensation, channel estimation, and I/Q imbalance compensation.

In OFDM systems, the term orthogonal dictates an exact mathematical relationship between the frequencies of the subcarriers. The orthogonality of the OFDM-based systems depends on the situation that transmitter and receiver work with the same frequency reference. If this doesn't happen, the orthogonality of subcarrier will be destroyed which leads to inter-carrier Interference (ICI). Strictly speaking, all of the OFDM communication systems are sensitive to the frequency synchronization in form of carrier frequency offset (CFO). CFO severely degrades the performance of the OFDM systems, therefore estimating and compensating the

CFO is a must. The CFO create a shift( $\delta f$ ) in received signal, this shift can be seen in figure 1 [4].

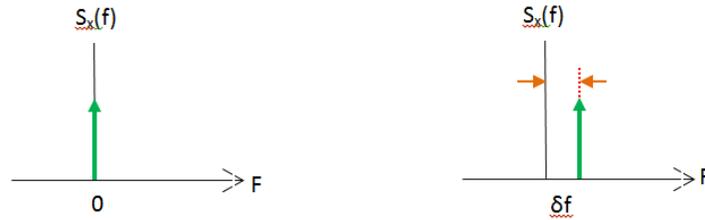


Figure1: Frequency offset ( $\delta f$ )

## 2. CHOICE OF OFDM PARAMETERS

The most important parameters that should be considered in any OFDM design are: guard time, symbol duration, the number of the subcarriers, subcarrier spacing, and type of the modulation. However, what dictates and assigns the selection of the parameters are the system requirements. Some of the common system requirements are: desired bit rate, available bandwidth (BW), Doppler values and acceptable delay spread. It is necessary to mention that some of these requirements can be conflicted each other. For example if we want to have a good delay spread tolerance, we should select a huge number of the subcarrier with the slight subcarrier spacing but for having a good tolerance against the phase noise the opposite is true. Therefore, there are different parameters for OFDM systems, which selecting them always leads to a tradeoff between them. Basically the three main ones that should be considered are: bit rate, bandwidth (BW), and delay spread. Since the guard time must be two to four times the Root Mean Squared of the Delay Spread (RMSDS) therefore among the mentioned parameters, the delay spread is the one that assign the guard time. Anyway, this value is related to the kind of the coding and modulation. It is worth to mention that the higher order of the QAM (i.e. 64-QAM) compare with QPSK has a higher degree of sensitivity to the ICI and ISI. The other prerequisite which has a direct impact on the selection of the parameters is the desired integer number of samples within the FFT/IFFT interval and in the symbol interval. The accepted OFDM time related and rate related parameters in IEEE 802.11 are respectively listed in table 1 and 2 [5].

Parameters	Values
Number of data sub-carriers	48
Number of pilot sub-carriers	4
Total number of sub-carriers	52
Sub-carrier frequency spacing	0.3125 MHz
IFFT/FFT period	3.2 $\mu$ s $(1/\Delta_f)$
Preamble duration	16 $\mu$ s
Signal Duration BPSK-OFDM symbol	4 $\mu$ s $(T_{GI}+T_{FFT})$
Guard Interval (GI) duration	0.8 $\mu$ s $(T_{FFT}/4)$
Training symbol GI duration	1.6 $\mu$ s $(T_{FFT}/2)$
Symbol interval	4 $\mu$ s $(T_{GI}+T_{FFT})$
Short training sequence duration	8 $\mu$ s $(10T_{FFT}/4)$
Long training sequence duration	8 $\mu$ s $(T_{GI}+T_{FFT})$

Table 1: The timing related parameters for the OFDM systems [5]

Modulation	Coding rate (R)	Coded bits per subcarrier ( $N_{\text{BPSK}}$ )	Coded bits per OFDM symbol ( $N_{\text{CBPS}}$ )	Data bits per OFDM symbol ( $N_{\text{DBPS}}$ )	Data rate (Mb/s) (20 MHz channel spacing)	Data rate (Mb/s) (10 MHz channel spacing)	Data rate (Mb/s) (5 MHz channel spacing)
BPSK	1/2	1	48	24	6	3	1.5
BPSK	3/4	1	48	36	9	4.5	2.25
QPSK	1/2	2	96	48	12	6	3
QPSK	3/4	2	96	72	18	9	4.5
16-QAM	1/2	4	192	96	24	12	6
16-QAM	3/4	4	192	144	36	18	9
64-QAM	1/2	6	288	192	48	24	12
64-QAM	3/4	6	288	216	54	27	13.5

Table 2: The rate dependent parameters for the OFDM systems [5]

As you see in Table 2 data rate has different values, these values vary according to the type of the modulation and has a different value for BPSK, QPSK, 16QAM, and 64QAM. However the theoretical values are calculated as follows:

$$\frac{E_b}{N_0} = \frac{W N_g}{R N_s} 10^{\frac{SNR}{10}} \quad (1)$$

$$BER = \frac{1}{2} erfc(\sqrt{E_b/N_0}) \quad (2)$$

Where  $w$  is; signal bandwidth (BW),  $N_s$  is symbol length,  $R$  is data rate and  $N_g$  is guard interval length.

### 3. THE ORIGIN OF CFO

Two factors can introduce the frequency offset; the first one is the lack of the matching carrier frequency between the receiver oscillator and transmitter oscillator and the second one is due to the Doppler shift. The Doppler shift can be created when there is a relative motion between the transmitter and receiver or both of them. The Doppler Effect is given as follows [4]:

$$f_d = \frac{v f_c}{c} \quad (3)$$

$f_d$ ,  $c$  and  $v$  are respectively Doppler frequency, the speed of light, and the velocity of the moving receiver (i.e. 100 km/h). The normalized CFO ( $\epsilon$ ) is:

$$\epsilon = \frac{f_{\text{offset}}}{\Delta f} \quad (4)$$

$\Delta f$  is the subcarrier spacing, the  $\epsilon$  has two portions, one integer ( $\epsilon_i$ ) and one fractional ( $\epsilon_f$ ) so:

$$\epsilon = \epsilon_i + \epsilon_f \quad (5)$$

The instability of the oscillators is the one that can cause the mismatch between the transmitter and the receiver oscillator. The factors that can cause this instability are: temperature, humidity, the interference of the other magnetic field (which these factors are categorized as the working environment), the tolerance of the electronic elements and aging. Aging is the frequency change with time while the other factors i.e. working environment, power supply and etc. will be kept

constant. By the way, two important factors for oscillator performance are stability and accuracy. Stability shows how well an oscillator can produce the same frequency in a specified interval of time, but the accuracy indicates the offset from the ideal value. Although the frequency errors introduced by the frequency mismatch between the local oscillator in the transmitter and the receiver, nonetheless this frequency mismatch due to the electronics elements tolerances are avoidable. So at all the time there is a difference between the carrier frequency which has been produced in the receiver side, with the one that has been generated in the transmitted side. This frequency offset is defined as follows [4]:

$$f_{offset} = f_c - f'_c \quad (6)$$

$f_c$  and  $f'_c$  are respectively carrier frequency in the transmitter and the carrier frequency in receiver. Figure 2 illustrates the impact of the frequency offset. With the presence of the frequency offset, the sampling cannot be exactly done at the center frequencies of the subcarriers therefore the amplitude of the desired subcarriers will be decreased.

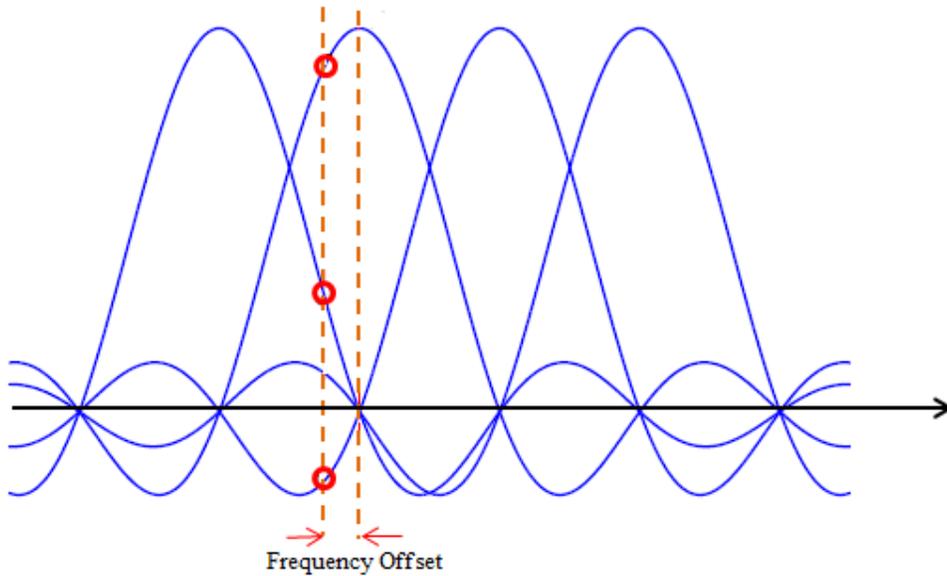


Figure2: The impact of frequency offset

#### 4. CFO ESTIMATION ALGORITHM AND TECHNIQUES

Since the CFO leads to Inter-carrier Interference (ICI) therefore its degradation on OFDM systems is more than the effect of noise. However, the number of numerous proposed CFO estimation techniques and algorithm is an evidence of its importance. However these various techniques can usually be classified into two major branches:

1. Training based algorithm
2. Blind algorithm and Semi-blind algorithm

In training based algorithm, the training sequences are sent with the transmitted signal, which receiver can use them. This training sequences cause to limit the number of computation at

receiver, therefore these algorithms have a low computational complexity. In contrast, since the training sequences are needed to be sent during the signal transmission, it will lead to the reduction of the effectiveness of the data throughput.

Blind CFO estimation algorithm is the other one that is used for CFO estimation. In these methods, the statistical properties of the received signal are used for estimating the CFO. These algorithms do not need to any kind of the training sequences, thus it will not have any training overhead. on the other hand, nothing will be send with the transmitted signal that the receiver can use it for estimating CFO, therefore the receiver will not have any knowledge base of the transmitted signal, so it has to start from scratch, therefore these algorithms compare with the trained based has the higher computational complexity. Figure 3 shows the block diagram of a typical OFDM receiver with frequency synchronization.

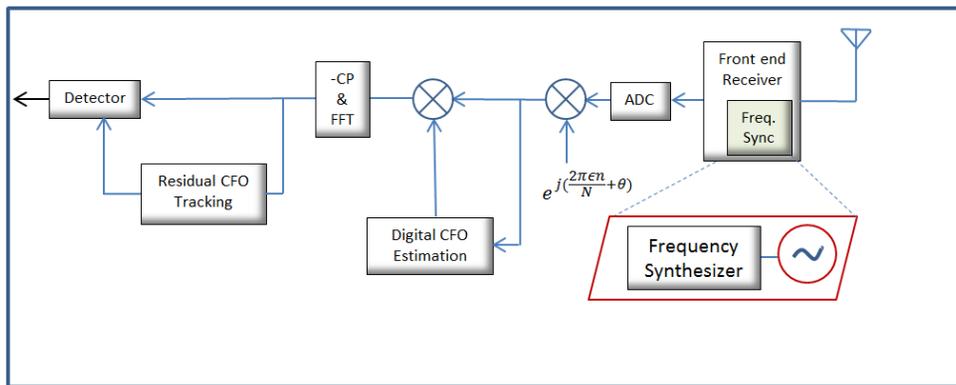


Figure 3: The block diagram of a typical OFDM receiver with frequency synchronization

## 5. MODEL FOR CFO

Figure 4 illustrates the block diagram of the OFDM wireless transceiver system. Let's consider this figure with N subcarriers with the sampling rate  $1/T_s$

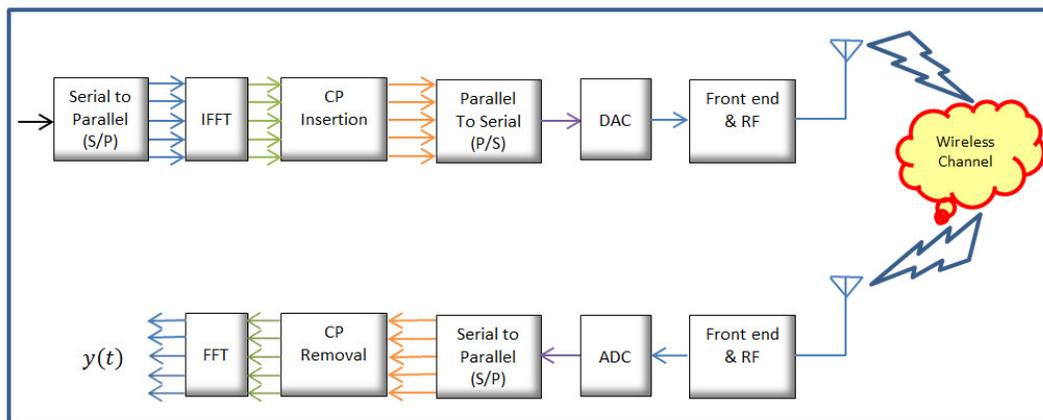


Figure 4: block diagram of the OFDM wireless transceiver system [4]

Therefore we have:

$$X(n) = W_p S(n) \quad (7)$$

$$y(n) = [y_0(n) y_1(n) y_2(n) \dots y_{N-1}(n)]^T \quad (8)$$

$$y(n) = W_p H s(n) \quad (9)$$

In where

$$H = \begin{bmatrix} H(1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & H(P) \end{bmatrix} \quad (10)$$

After DFT unit we have:

$$W_p^H y(n) = H [s_1(n) \dots s_p(n)]^T \quad (11)$$

$$y(n) = E W_p H s(n) e^{j\phi((N+L)(n-1))} \quad (12)$$

$$E = \text{diag}(1, e^{j\phi}, \dots, e^{j(n-1)\phi}) \quad (13)$$

In where,  $E$  is carrier offset matrix,  $W_p$  is a  $N \times P$  matrix,  $s(n)$  is data,  $H$  is channel response and  $L$  is length of cyclic prefix.

$$W_p^H y(n) = W_p^H E W_p H s(n) e^{j\phi(N+L)(n-1)} \quad (14)$$

Here the CFO and  $\phi$  are needed to be estimated and applied.

## 6. THE EFFECT OF CFO ON THE RECEIVED SIGNAL

A received signal without carrier frequency offset can be considered as follows:

$$r[n] = H[n]X(n) + \omega[n] \quad \text{where } 0 \leq n \leq N-1 \quad (15)$$

The equation 15 can be considered if and only if the effect of CFO is ignored. In the presence of the CFO The signal can be given as [6]:

$$r[n] = H[n]X(n) \frac{\sin(\pi\varepsilon)}{N \sin(\frac{\pi\varepsilon}{N})} e^{\frac{j\pi\varepsilon(N-1)}{N}} + \sum_{m=0, m \neq n}^{N-1} H[m]X(m) \frac{\sin[\pi(m-n+\varepsilon)]}{N \sin[\frac{\pi(m-n+\varepsilon)}{N}]} e^{\frac{j\pi(m-n+\varepsilon)(N-1)}{N}} + \omega[n] \quad (16)$$

Where the normalized frequency offset ( $\varepsilon$ ) is:

$$\varepsilon = \frac{f_0}{\Delta f} \quad (17)$$

The second part of the equation 16 shows the effect of the ICI.

As it can be seen in the equation 16, the amplitude will be decreased by the factor:

$$\frac{\sin(\pi\varepsilon)}{N \sin(\frac{\pi\varepsilon}{N})} \quad (18)$$

and the signal experiences the phase shift equal to:

$$\theta = e^{j\pi\varepsilon(N-1)/N} \quad (19)$$

On the other hand, since the adjacent subcarriers cannot be sample at their zero crossing, therefore they cause interference.

### 7. SENSITIVITY TO FREQUENCY OFFSET

When there is the different integer number of cycles in each FFT interval the orthogonality between the subcarrier will be kept, otherwise the subcarriers will not be orthogonal. Therefore we can conclude that the frequency offset will happen when the number of cycle in FFT interval will not be an integer.

In the center of the OFDM spectrum the quantity of the Inter-Carrier Interference (ICI), in compare with their amount at the band edges, is almost double. This increase comes from the interfering of the subcarriers from both sides on the middle ones. However the degradation of the SNR which is introduced by the frequency offset can be given by [7]:

$$D_{freq} \cong \frac{10}{3 \ln 10} (\pi \Delta f T)^2 \frac{E_s}{N_0} \tag{20}$$

Where  $D_{freq}$ ,  $T$ ,  $E_b$  and  $N_0$  are in order; frequency offset, symbol duration, energy per bit (for OFDM signal) and one sided noise Power Spectrum Density (PSD).

Figure 5 illustrates the SNR degradation as a function of the frequency offset to the subcarrier spacing. For this illustration we consider two different values for  $E_s/N_0$  which are 18 and 16 db. However the maximum acceptable frequency offset can only happen when the frequency offset is less than one percent of the subcarrier space. As a result, for overcoming to the mentioned problem the frequency synchronization must be used before the FFT.

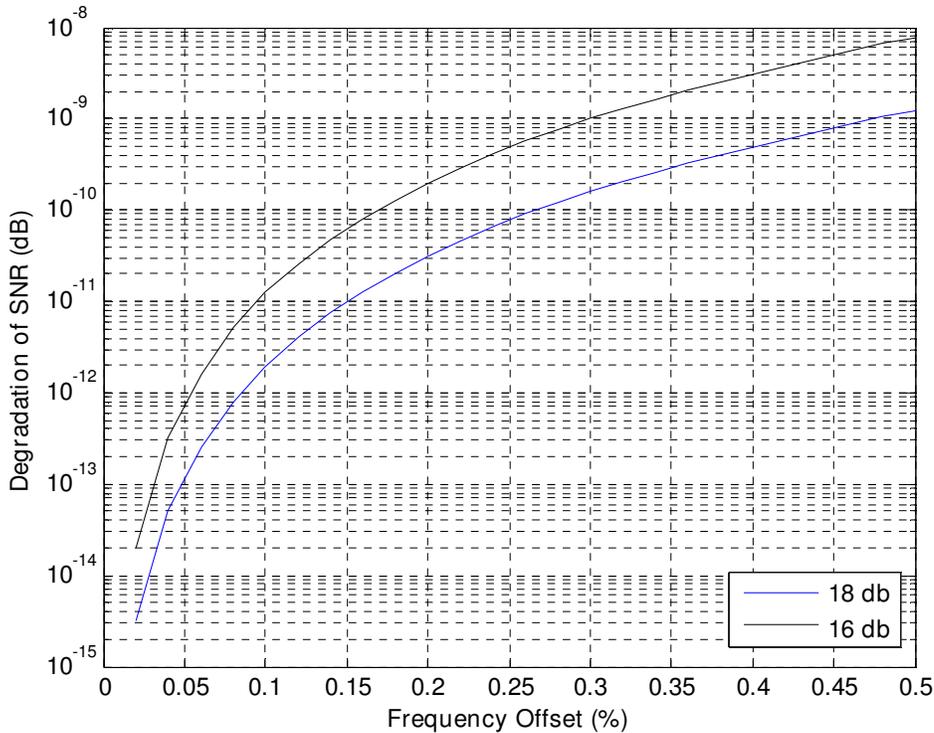


Figure 5: SNR degradation as a function of the frequency offset

## 8. THE EFFECT OF INTEGER AND FRACTIONAL PART OF CFO

As we have previously mentioned the normalized CFO has made of two parts: integer part ( $\varepsilon_i$ ) and fractional part ( $\varepsilon_f$ ). The integer part causes that the received signal to the receiver experiences a cyclic shift. Now let's see the effect of the fractional part. The received signal in time domain can be considered as follows [8]:

$$Y_l[k] = \sum_{n=0}^{N-1} y_l[n] e^{-j2\pi kn/N} = H_l[k]X_l[k] + Z_l[k] \quad (21)$$

$Y_l[k]$ ,  $H_l[k]$ ,  $X_l[k]$  and  $Z_l[k]$  represent the received symbol, channel frequency response, transmitted symbol, and the noise in the frequency domain for the  $k^{\text{th}}$  subcarrier.

$$IDFT\{Y_l[k]\} = y_l[n] = \frac{1}{N} \sum_{k=0}^{N-1} H[k]X_l[k] e^{2j\pi(k+\varepsilon)(n+\delta)/N} + Z_l[n] \quad (22)$$

Where  $Z_l[n] = IDFT\{Z_l[k]\}$ . To see the effect of the CFO, we discard the phase noise and merely consider the CFO [9], therefore the equation 22 can be written as:

$$y_l[n] = \frac{1}{N} \sum_{k=0}^{N-1} H[k]X_l[k] e^{2j\pi(k+\varepsilon)/N} + Z_l[n] \quad (23)$$

The FFT of the equation 23 yields [8]:

$$y_l[k] = FFT\{y_l(n)\} \quad (24)$$

$$y_l[k] = \frac{\sin\pi\varepsilon_f}{N\sin(\pi\varepsilon_f/N)} e^{j\pi\varepsilon_f(N-1)/N} H_l[k]X_l[k] + I_l[k] + Z_l[k] \quad (25)$$

The first part of the equation 25 shows the effect of the Fractional part of the CFO. The figures 6 and 7 show its effect. However as we expected; by increasing the fractional part of the CFO the effect of it on phase distortion and amplitude increases. This can be seen in figures 6 and 7. In these figures the effect of noise and STO are not considered at all.

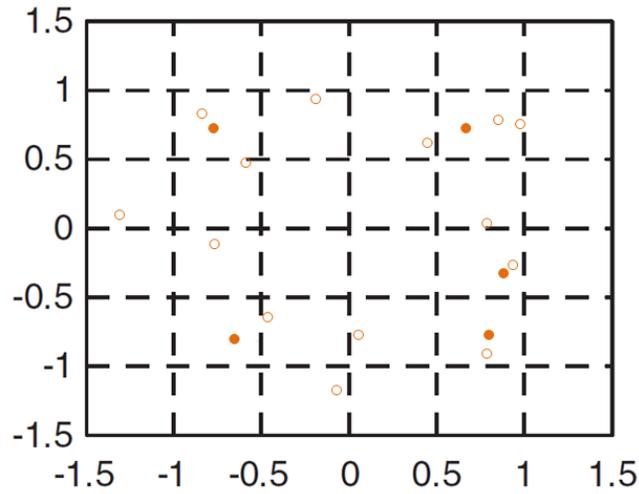


Figure 6: The effect of CFO when  $\varepsilon = 0.4$

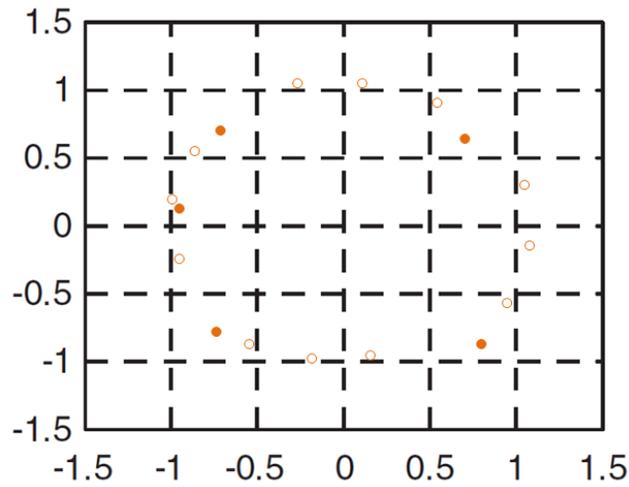


Figure 7: The effect of CFO when  $\varepsilon = 0.06$

### 9. THE EFFECT OF CFO ON PHASE SHIFT

The figure 8 illustrates the effect of CFO on the phase in the time domain and figure 9 presents the phase change between them. Here we consider the system is not exposing to any noise. The solid line in figure 8 shows when there is no CFO (i.e. perfect case), and dash lines shows when there is CFO.

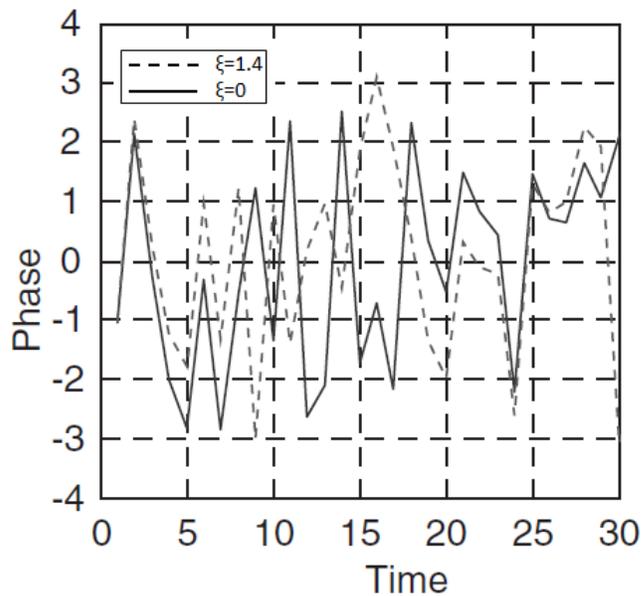


Figure 8: The effect of CFO ( $\xi$ ) on phase in the time domain

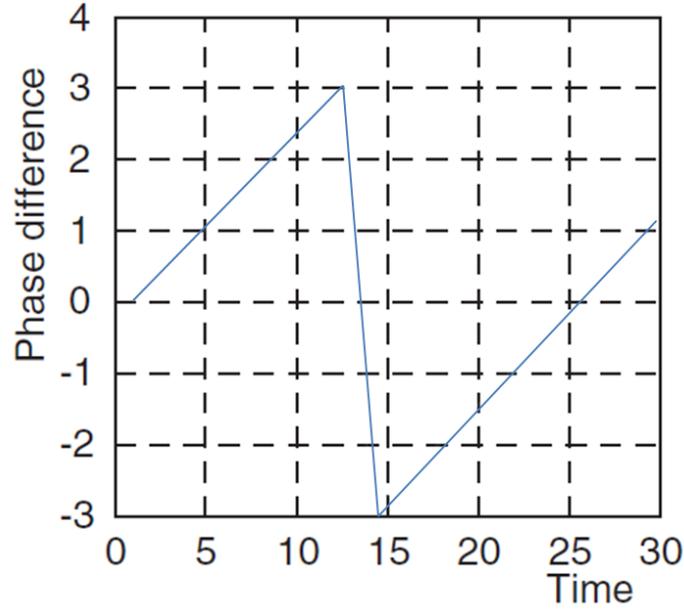


Figure 9: The effect of CFO ( $\xi$ ) on phase difference

## 10. ESTIMATING CFO

The algorithms that use CP for estimating CFO, can only estimate the CFO in a limited range. This limited range is:

$$-\frac{1}{2} \leq \varepsilon \leq \frac{1}{2} \quad (26)$$

To increase this limit, it is needed to decrease the distance between two blocks of samples for correction, which this can be done by using training symbol. To increase the CFO estimation within a bigger range we can use equation 27 [10, 11].

$$\hat{\varepsilon} = \frac{D}{2\pi} \arg \left\{ \sum_{n=0}^{N/D-1} y_i^*[n] y_i[n + N/D] \right\} \quad (27)$$

Where  $N/D$  is an integer; Here the range is:

$$-\frac{D}{2} \leq \varepsilon \leq \frac{D}{2} \quad (28)$$

The CFO estimation range by using equation 27 will be increased to  $|\varepsilon| \leq D$ . However this increase cost to the degradation of the MSE performance. Figure 10 shows the effect of increasing the estimation range of CFO versus the MSE performance for  $D=1$  (red graph) and  $D=5$  (black graph).

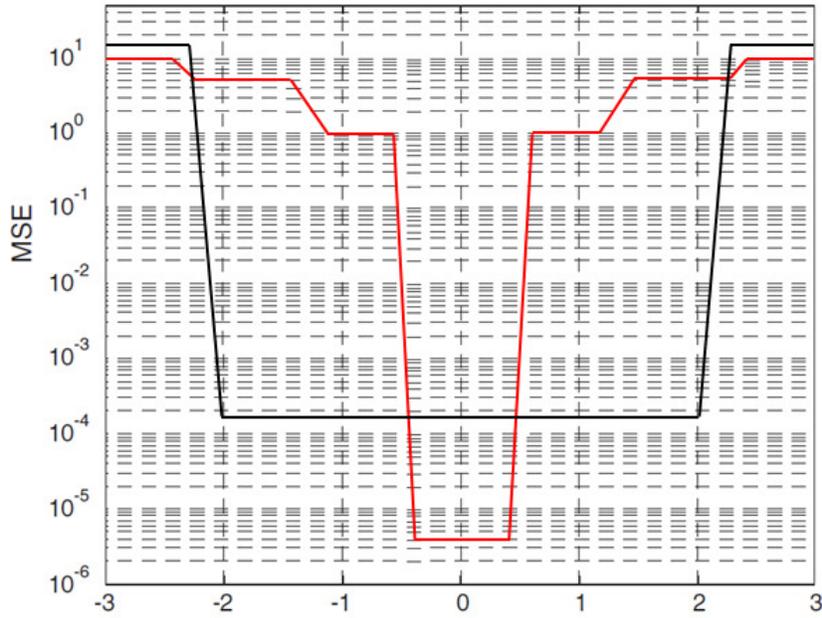


Figure10: CFO VS the MSE for D=1 (Red graph) and D=5 (Black Graph)

## 11. DEGRADATION OF OFDM SYSTEMS DUE TO THE CFO

The complex transmitted OFDM signal in a period T at the frequency  $m/T$  can be state as [12]:

$$s(t) = \left( \sum_{m=0}^{N-1} a_m e^{\frac{j2\pi mt}{T}} \right) e^{j\theta(t)} \quad (29)$$

In which  $a_m$  is data symbol and the term  $\theta(t)$  represents the time varying phase due to the carrier frequency offset between transmitter and receiver.

With an accepted approximation the degradation in terms of dB is defined as follows [12]:

$$D \cong \frac{10}{\ln 10} \left( (1 - E_0^2) + V_0 \frac{E_s}{N_0} \right) \quad (30)$$

In which, the signal to noise ratio (SNR) is:  $E_s/N_0$ , and  $V_0$  is the variance for others noises and  $E_0^2$  is the power of the component.

In case of the presence of the frequency offset ( $\Delta f$ ) between the transmitter and receiver, the  $\theta(t)$  in equation29 will be defined as follows:

$$\theta(t) = 2\pi\Delta f t + \theta_0 \quad (31)$$

Therefore the degradation (D) for OFDM is defined as [12]:

$$D \cong \frac{10}{3Ln 10} \left( \pi N \frac{\Delta f}{R} \right)^2 \frac{E_s}{N_0} \quad (32)$$

From equation 32, it is obvious that the degradation for an OFDM is proportional with the square root of the frequency offset and it is also proportional with  $E_s/N_0$ . In another word, OFDM is very sensitive to frequency offset; the frequency offset can cause degradation in OFDM systems.

## 12. CONCLUSIONS

In this paper a brief framework for carrier frequency offset (CFO) is provided. The importance of the study of the carrier frequency offset estimation and the source of the CFO for OFDM systems has been reviewed. We showed the CFO causes the received signal rotates quicker in time domain when the phase differences increase linearly. We have also showed; increasing the estimation range leads to decrease the MSE performance. However it is shown that the system performance can severely be affected by CFO. Further study can be carried out in this area by considering the other parameters which have the impact on degrading the performance of the OFDM systems.

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