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High dimensional real parameter optimization with teaching learning based optimization

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A B S T R A C T

In this paper, a new optimization technique known as Teaching–Learning-Based Optimization (TLBO) is implemented for solving high dimensional function optimization problems. Even though there are several other approaches to address this issue but the cost of computations are more in handling high dimensional problems. In this work we simulate TLBO for high dimensional benchmark function optimizations and compare its results with very widely used alternate techniques like Differential Evolution (DE) and Particle Swarm Optimization (PSO). Results clearly reveal that TLBO is able to address the computational cost issue for all simulated functions to a large dimensions compared to other two techniques.

1. Introduction

Even though there is a huge amount of work dealing with global optimization, there are still not many powerful techniques to be used for dense high-dimensional functions. One of the main reasons is the high-computational cost involved. Usually, the approaches are computationally expensive to solve the global optimization problem reliably. Very often, it requires many function evaluations and iterations and arithmetic operations within the optimization code itself. For practical optimization applications, the evaluation of f is often very expensive to compute and large number of function evaluations might not be very feasible. In recent past, there is growing demand in using evolutionary computation techniques for solving global function optimization problems. Among them, Genetic Algorithm (Holland, 1975) Particle Swarm Optimization (PSO) (Kennedy & Eberhart, 1995) ,Differential Evolution (DE) (Storn & Price, 1997; Price et al., 2005) and Artificial Bee Colony (ABC) (Karaboga & Basturk, 2007; Karaboga & Akay, 2009) etc. are widely used ones. These techniques and its several variants have been implemented for many benchmark global constrained and unconstrained function optimizations (Karaboga & Basturk, 2007; Karaboga & Akay, 2009; Mezura-Montes & Coello, 2005; Zavala et al., 2005; Becerra & Coello, 2006; Huang et al., 2007). However, it remains as a great

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challenge to solve high dimensional problems with reasonably less function evaluations. Recently, a new optimization techniques based on teaching learning approach known as Teaching Learning based optimization (TLBO) (Rao et al., 2011) is reported to produce better results as regard to the convergence speed. Rao, et al (Rao et al., 2012) have simulated many constrained and unconstrained real parameter optimization problems. Rao and Patel (2012) described the elitist TLBO algorithm for solving complex constrained optimization problems. The authors had provided the complete details of TLBO algorithm regarding its algorithm-specific parameter-less concept and the effect of common control parameters such as elite size, population size and number of generations on the performance of the algorithm. The points raised by Matej Črepinšek et al. (2012) were already addressed by Rao and Patel (2012).

In this paper we have attempted to simulate TLBO for various benchmark problems with different dimensions ranging from 10 to 500 to establish the effectiveness of TLBO over very popular classical PSO and DE.

The rest of the paper is organized as follows. In section 2 , we explain TLBO in brief. Section 3 provides the benchmark functions, parameter settings and simulation results. The conclusion and future enhancement is discussed in Section 4.

2. Teaching–learning-based optimization

This optimization method is based on the effect of the influence of a teacher on the output of learners in a class. It is a population based method and like other population based methods it uses a population of solutions to proceed to the global solution. A group of learners constitute the population in TLBO. In any optimization algorithms there are numbers of different design variables. The different design variables in TLBO are analogous to different subjects offered to learners and the learners' result is analogous to the 'fitness', as in other population-based optimization techniques. As the teacher is considered the most learned person in the society, the best solution so far is analogous to Teacher in TLBO. The process of TLBO is divided into two parts. The first part consists of the 'Teacher Phase' and the second part consists of the 'Learner Phase'. The 'Teacher Phase' means learning from the teacher and the 'Learner Phase' means learning through the interaction between learners. In the sub-sections below we briefly discuss the implementation of TLBO.

2.1 Initialization

Following are the notations used for describing the TLBO:
N: number of learners in a class i. e. "class size"
D: number of courses offered to the learners
$MAXIT$: maximum number of allowable iterations
The population X is randomly initialized by a search space bounded by matrix of N rows and D columns. The jth parameter of the ith learner is assigned values randomly using the equation

$$x^0_{(i,j)} = x_j^{min} + rand \times \left(x_j^{max} - x_j^{min}\right), \tag{1}$$

where $rand$ represents a uniformly distributed random variable within the range (0, 1), x_j^{min} and x_j^{max} represent the minimum and maximum value for jth parameter. The parameters of ith learner for the generation g are given by

$$X^g_{(i)} = [x^g_{(i,1)}, x^g_{(i,2)}, x^g_{(i,3)}, \dots\dots, x^g_{(i,j)}, \dots\dots, x^g_{(i,D)}] \tag{2}$$

2.2 Teacher Phase

The mean parameter M^g of each subject of the learners in the class at generation g is given as

$$M^g = [m_1^g, m_2^g, \dots \dots, m_j^g, \dots \dots, m_D^g] \quad (3)$$

The learner with the minimum objective function value is considered as the teacher $X_{Teacher}^g$ for respective iteration. The Teacher phase makes the algorithm proceed by shifting the mean of the learners towards its teacher. To obtain a new set of improved learners a random weighted differential vector is formed from the current mean and the desired mean parameters and added to the existing population of learners.

$$Xnew_{(i)}^g = X_{(i)}^g + rand \times (X_{Teacher}^g - T_F M^g) \quad (4)$$

T_F is the teaching factor which decides the value of mean to be changed. Value of T_F can be either 1 or 2. The value of T_F is decided randomly with equal probability as,

$$T_F = round[1 + rand(0,1)\{2 - 1\}] \quad (5)$$

It may be noted here that T_F is not a parameter of the TLBO algorithm. The value of T_F is not given as an input to the algorithm and its value is randomly decided by the algorithm using Eq. (5). After conducting a number of experiments on many benchmark functions it is concluded that the algorithm performs better if the value of T_F is between 1 and 2. However, the algorithm is found to perform much better if the value of T_F is either 1 or 2 and hence to simplify the algorithm, the teaching factor is suggested to take either 1 or 2 depending on the rounding up criteria given by Eq.(5).
If $Xnew_{(i)}^g$ is found to be a superior learner than $X_{(i)}^g$ in generation g , than it replaces inferior learner $X_{(i)}^g$ in the matrix.

2.3 Learner Phase

In this phase the interaction of learners with one another takes place. The process of mutual interaction tends to increase the knowledge of the learner. The random interaction among learners improves his or her knowledge. For a given learner$X_{(i)}^g$, another learner $X_{(r)}^g$ is randomly selected$(i \neq r)$. The i^{th} parameter of the matrix $Xnew$ in the learner phase is given as

$$Xnew_{(i)}^g = \begin{cases} X_{(i)}^g + rand \times \left(X_{(i)}^g - X_{(r)}^g\right) & if\ f\left(X_{(i)}^g\right) < f(X_{(r)}^g) \\ X_{(i)}^g + rand \times \left(X_{(r)}^g - X_{(i)}^g\right) & oterwise \end{cases} \quad (6)$$

D Algorithm Termination

The algorithm is terminated after $MAXIT$ iterations are completed.
Details of TLBO can be refereed in (Rao et al., 2012)

3. Benchmark Functions, parameter settings and Simulation results

In this work we have simulated two classes of functions: Unimodal and Multimodal. Further, we have chosen separable and un-separable functions in both of these classes. In the Table 1 the descriptions of all simulated functions are presented.

Table 1

Benchmark functions: C: Characteristic, U: Unimodal, M:Multimodal, S:Separable , N:Non-Separable

No.	Function	C	Range	Formulation	Value
f_1	Sphere	US	[-100,100]	$f(x)=\sum_{i=1}^{D} x_i^2$	$f_{\min}=0$
f_2	SumSquares	US	[-10,10]	$f(x)=\sum_{i=1}^{D} ix_i^2$	$f_{\min}=0$
f_3	Quartic	US	[-1.28,1.28]	$f(x)=\sum_{i=1}^{D} ix_i^4+rand(0,1)$	$f_{\min}=0$
f_4	Zakharov	UN	[-5,10]	$f(x)=\sum_{i=1}^{D} x_i^2+(\sum_{i=1}^{D}0.5ix_i)^2+(\sum_{i=1}^{D}0.5ix_i)^4$	$f_{\min}=0$
f_5	Schwefel 1.2	UN	[-100,100]	$f(x)=\sum_{i=1}^{D}(\sum_{j=1}^{i}x_j)^2$	$f_{\min}=0$
f_6	Schwefel 2.22	UN	[-10,10]	$f(x)=\sum_{i=1}^{D}\lvert x_i\rvert+\prod_{i=1}^{D}\lvert x_i\rvert$	$f_{\min}=0$
f_7	Schwefel 2.21	MN	[-100,100]	$f(x)=\max_i\left\{\lvert x_i\rvert,1\le i\le D\right\}$	$f_{\min}=0$
f_8	Rastrigin	MS	[-5.12,5.12]	$f(x)=\sum_{i=1}^{D}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	$f_{\min}=0$
f_9	Griewank	MN	[-600,600]	$f(x)=\frac{1}{4000}\sum_{i=1}^{D}x_i^2-\prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}})+1$	$f_{\min}=0$
f_{10}	Ackley	MN	[-32,32]	$f(x)=-20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{D}\cos(2\pi_i x_i\right)+20+e$	$f_{\min}=0$
f_{11}	Noncontinuous Rastrigin	MN	[-5.12,5.12]	$f(x)=\sum_{i=1}^{D}\left[y_i^2-10\cos(2\pi y_i)+10\right],\quad \text{where } y_i=\begin{cases} x_i & \lvert x_i\rvert<0.5 \\ \frac{round(2x_i)}{2} & \lvert x_i\rvert\ge 0.5\end{cases}$	$f_{\min}=0$
f_{12}	Multimod	MN	[-10,10]	$f(x)=\sum_{i=1}^{D}\lvert x_i\rvert\prod_{i=1}^{D}\lvert x_i\rvert$	$f_{\min}=0$

Parameter settings

In all simulations of our work, the values of the common parameters used in each algorithm such as population size and total evaluation number were chosen to be the same. Population size is 50 and the maximum number fitness function evaluation is fixed as 100,000 for all functions. The other specific parameters of algorithms are given below:

PSO Settings: Cognitive and Social components, c_1, c_2 are constants that can be used to change the weighting between personal and population experience, respectively. In our experiments cognitive and social components are both set to 2 (Kennedy & Eberhart, 1995). Inertia weight, which determines how the previous velocity of the particle influences the velocity in the next iteration, is 0.5 (Kennedy & Eberhart, 1995).

DE Settings: In DE, F is a real constant which affects the differential variation between two Solutions and set to F = 0.5*(1+ rand (0, 1)) where rand (0, 1) is a uniformly distributed random number within the range [0, 1]. In our simulation the value of crossover rate, which controls the change of the diversity of the population, is chosen to be R = (Rmax – Rmin) * (MAXIT–*iter*) / MAXIT where Rmax=1 and Rmin=0.5 are the maximum and minimum values of scale factor R, *iter* is the current iteration number and MAXIT is the maximum number of allowable iterations as recommended in (Storn & Price, 1997).

TLBO Settings: For TLBO there is no such constant to set.

We have simulated each function with different dimensions for each algorithm. The range of dimensions is chosen from 10 to 500. The simulated results are presented in Table 2 to Table 5.

Table 2
Global Minimum values of Unimodal functions

Dimension/Function	Algo	Global Min	f_1	f_2	f_3	f_4	f_5	f_6
Dim=10	PSO	Mean	2.3401e-237	2.0354e-183	0.0089	8.9128e-38	1.2268e-124	2.4429e-097
		Std	0	1.0379e-184	0.0031	3.9324e-36	2.8240e-132	1.1898e-098
	DE	Mean	1.0063e-094	4.1131e-72	0.0023	2.2662e-05	7.0651e-075	3.0593e-052
		Std	8.7780e-095	1.4734e-72	0.0011	1.6209e-05	1.3162e-076	1.1798e-053
	TLBO	Mean	0	0	1.9695e-04	4.3766e-248	7.3945e-242	4.8330e-185
		Std	0	0	4.5003e-05	0	0	0
Dim=20	PSO	Mean	3.1110e-061	8.1330e-53	0.1592	15.8510	5.2511e-37	3.9894e-33
		Std	2.4790e-061	1.0128e-54	0.0092	7.8248	6.3614e-41	1.9909e-34
	DE	Mean	5.7731e-33	3.6391e-26	0.0146	4.8033	3.3522e-025	4.5411e-023
		Std	2.4297e-34	1.8754e-26	0.0045	0.8252	1.3125e-025	1.2248e-023
	TLBO	Mean	0	0	2.7113e-04	8.8131e-252	6.3842e-242	1.1298e-186
		Std	0	0	2.8977e-05	0	0	0
Dim=30	PSO	Mean	8.2188e-021	1.6652e-14	0.6991	90.7945	3.2148e-13	2.1028e-13
		Std	2.9862e-021	1.0008e-14	0.0162	50.1516	8.6861e-15	1.9843e-13
	DE	Mean	6.4822e-016	3.7367e-12	0.0459	63.3415	2.2970e-010	4.1134e-12
		Std	4.2132e-016	1.1823e-12	0.0129	9.1117	6.6788e-011	1.0519e-12
	TLBO	Mean	0	8.6471e+04	3.9089e-04	2.2984e-249	4.6496e-242	1.4649e-185
		Std	0	478.8023	1.2227e-05	0	0	0
Dim=50	PSO	Mean	1.2286e-004	0.0055	3.6213	401.4852	108.5661	0.0061
		Std	1.0186e-004	0.0045	0.5123	60.1418	100.1911	0.0016
	DE	Mean	3.4331e-04	4.2656e-04	0.2836	309.6465	19.8894	7.8218e-05
		Std	2.4801e-04	2.3197e-04	0.1662	38.5938	5.2115	1.2227e-05
	TLBO	Mean	0	0	1.8051e-04	3.2345e-249	8.7479e-241	6.0080e-188
		Std	0	0	2.1638e-05	0	0	0
Dim=100	PSO	Mean	1.1921e+04	3.5359e+03	58.1882	940.1946	4.6916e+09	14.4230
		Std	1.0696e+04	2.4055e+03	5.1219	20.0986	1.6991e+09	4.4230
	DE	Mean	510.0908	16.7022	6.4277	1.0510e+03	1.6602e+08	1.1590
		Std	51.5882	0.9354	1.1227	761.6408	1.1192e+06	0.2612
	TLBO	Mean	0	0	2.5271e-04	1.7649e-250	4.4493e-244	3.9209e-188
		Std	0	0	4.3937e-05	0	0	0
Dim=150	PSO	Mean	5.5640e+04	6.2961e+04	215.1159	1.7563e+03	1.9115e+011	133.1091
		Std	2.0184e+04	4.0591e+04	30.1552	2.1431e+02	1.7021e+011	7.1091
	DE	Mean	1.2211e+04	132.4149	103.3661	1.4574e+03	3.3339e+010	32.8534
		Std	2.6112e+03	0.9614	6.3661	0.3151	1.2010e+09	2.2033
	TLBO	Mean	0	0	3.1182e-04	4.9813e-252	1.2919e-242	1.0194e-187
		Std	0	0	1.8591e-05	0	0	0
Dim=200	PSO	Mean	8.4231e+04	1.5173e+05	551.1816	2.6138e+03	1.2816e+012	215.1191
		Std	5.6423e+04	2.1431e+04	52.0612	0.2017e+03	1.1992e+012	12.9101
	DE	Mean	4.4889e+04	625.0197	431.9708	2.3343e+03	5.1829e+011	125.4735
		Std	1.6261e+04	0.9916	10.1509	2.1338e+02	5.0912e+09	5.6251
	TLBO	Mean	0	0	2.1619e-04	8.1514e-253	5.6169e-242	2.4748e-188
		Std	0	0	8.3406e-05	0	0	0
Dim=300	PSO	Mean	2.6462e+05	3.1159e+05	3.4339e+03	3.1622e+03	2.5035e+013	546.5376
		Std	1.7996e+05	2.5112e+04	1.2672e+03	5.4300e+02	2.0355e+013	28.7489
	DE	Mean	1.5822e+05	2.0308e+03	2.2402e+03	3.6662e+03	1.5572e+013	422.2928
		Std	6.1221e+04	1.0039e+02	1.0012e+02	2.3132e+02	1.2150e+011	12.0367
	TLBO	Mean	0	0	2.2102e-04	3.2057e-252	3.0600e-241	3.1167e-186
		Std	0	0	5.5212e-05	0	0	0
Dim=400	PSO	Mean	4.5115e+05	7.0685e+05	5.1400e+03	4.1871e+09	1.7541e+014	805.9514
		Std	2.6001e+05	3.1129e+05	2.5316e+03	2.7055e+09	1.0019e+014	65.6746
	DE	Mean	3.1822e+05	3.6115e+03	6.1210e+03	5.0545e+03	1.2673e+014	768.9601
		Std	1.8700e+05	2.0019e+02	2.1159e+02	2.4560e+2	1.2513e+013	20.8514
	TLBO	Mean	0	0	2.1161e-04	3.0088e-251	3.9784e-242	4.6076e-188
		Std	0	0	2.9460e-05	0	0	0
Dim=500	PSO	Mean	5.1189e+05	9.7511e+05	8.4755e+03	8.7113e+15	6.6841e+014	1.3662e+03
		Std	3.8983e+05	3.1119e+05	1.1619e+03	2.6343e+13	2.4521e+014	1.0128e+03
	DE	Mean	4.6919e+05	5.2911e+03	1.2112e+04	6.2405e+03	5.3563e+014	2.8824e+11
		Std	1.1119e+05	1.0019e+03	139.1529	1.1030e+03	2.2593e+013	9.5972e+10
	TLBO	Mean	0	0	3.2930e-04	6.5824e-251	8.3269e-240	1.6805e-186
		Std	0	0	8.2426e-05	0	0	0

Table 3

Function Evaluations for Unimodal functions

Dimension/ Function	Algo	Global Min	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}
Dim=10	PSO	Mean	9.0360e-10	13.9294	0.1008	2.0133	14.0000	0
		Std	5.2190e-12	4.0012	0.0179	1.1551	6.0000	0
	DE	Mean	3.7124e-29	0	0	4.4409e-15	6.1038e+04	0
		Std	5.1460e-31	0	0	4.0117e-30	1.6360e+03	0
	TLBO	Mean	1.0648e-287	0		4.4409e-15	0	0
		Std	0	0		4.0117e-30	0	0
Dim=20	PSO	Mean	2.0070e-03	30.8437	0.0099	4.2335	45.0120	0
		Std	1.1579e-03	3.8235	0.0019	2.1266	7.7501	0
	DE	Mean	2.6688e-7	1.7449e-06	0	2.6734e-13	0.0046	1.0097e-290
		Std	1.0071e-7	6.4310e-07	0	1.0997e-13	0.0011	0
	TLBO	Mean	4.1992e-288	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=30	PSO	Mean	0.0156	65.8286	0.0144	10.7825	82.9134	0
		Std	0.0052	12.6568	0.0137	2.8765	14.4239	0
	DE	Mean	0.0434	6.4506	7.4023e-12	2.0571e-07	14.4359	6.4520e-228
		Std	0.0094	1.5867	1.4528e-12	7.7213e-8	1.0423	0
	TLBO	Mean	3.0154e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=50	PSO	Mean	65.9625	139.2653	1.0561	14.2636	150.8362	6.6639e-310
		Std	2.3144	7.2668	0.9861	0.3413	19.3266	0
	DE	Mean	4.6989	80.9887	8.3224e-04	2.9213e-02	54.6163	1.3271e-179
		Std	0.1279	12.2012	6.0820e-04	1.0129e-02	6.1154	0
	TLBO	Mean	5.3387e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=100	PSO	Mean	76.4219	306.6107	157.4211	16.7488	456.9036	2.3900e-282
		Std	1.9252	87.4262	51.7580	1.1539	98.8347	0
	DE	Mean	41.9212	440.9707	2.2401	0.0049	308.4290	1.4817e-118
		Std	0.9116	11.0371	0.6811	0.0016	12.5375	0
	TLBO	Mean	1.8258e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=150	PSO	Mean	85.7225	641.8881	750.5357	18.2005	746.3139	5.7507e-278
		Std	2.0108	34.6227	61.9606	0.5202	110.2351	0
	DE	Mean	59.4771	921.9111	58.0067	3.4694	680.4148	7.7559e-86
		Std	0.4111	12.8201	9.9828	0.1291	45.7660	2.1599e-88
	TLBO	Mean	3.7261e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=200	PSO	Mean	85.1136	1.2415e+03	1.3672e+03	18.5129	1.2025e+03	3.4823e-264
		Std	1.4544	1.1749e+02	1.0071e+03	0.0829	2.5923e+02	0
	DE	Mean	69.6717	1.5051e+03	230.6390	12.6657	1.2266e+03	3.3532e-57
		Std	0.5158	16.8913	30.7851	0.1516	515.1192	1.4103e-58
	TLBO	Mean	4.7261e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=300	PSO	Mean	85.1305	2.2967e+03	1.4869e+03	18.7318	2.1001e+03	2.7262e-234
		Std	5.0775	1.1907e+02	765.9498	0.0717	1.9468e+03	0
	DE	Mean	77.9417	2.7209e+03	1.1549e+03	16.2115	2.3121e+03	7.7160e-28
		Std	0.8197	38.1902	1.0042e+03	0.3221	112.4358	3.0179e-29
	TLBO	Mean	2.2147e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=400	PSO	Mean	90.9604	3.6349e+03	3.8054e+03	18.9043	2.9384e+03	3.4563e-183
		Std	2.1917	3.9149e+02	1.0094e+03	0.0301	1.9212e+03	0
	DE	Mean	83.6613	3.9914e+03	2.4567e+03	17.6978	3.6283e+03	5.2065e+13
		Std	0.5443	23.1615	2.2903e+03	0.2119	111.5880	7.3173e+07
	TLBO	Mean	2.2584e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0
Dim=500	PSO	Mean	94.7888	4.4452e+03	5.0997e+03	19.1034	4.3197e+03	5.4208e-108
		Std	6.2757	1.0011e+3	73.1957	0.6034	2.1532e+02	2.1003e-124
	DE	Mean	86.1856	5.3880e+03	3.7707e+03	18.5342	4.9403e+03	1.2153e+43
		Std	0.9176	48.1192	1.0019e+03	0.1017	542.1129	7.1556e+10
	TLBO	Mean	3.0154e-287	0	0	4.4409e-15	0	0
		Std	0	0	0	4.0117e-30	0	0

Table 4
Global minimum values for Multimodal Functions

Dimension/Function	Algo	FEs	f_1	f_2	f_3	f_4	f_5	f_6
Dim=10	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64665	6.3238e+04	100,000	100,000	100,000	100,000
		Std	430.3467	8.7098e+03	0	0	0	0
Dim=20	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64945	64435	100,000	100,000	100,000	100,000
		Std	704.4624	750.5343	0	0	0	0
Dim=30	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64795	8.6471e+04	100,000	100,000	100,000	100,000
		Std	398.7805	478.8023	0	0	0	0
Dim=50	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64715	6.4909e+04	100,000	100,000	100,000	100,000
		Std	455.6801	291.9371	0	0	0	0
Dim=100	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	65065	1.3014e+03	100,000	100,000	100,000	100,000
		Std	261.3460	7.3653	0	0	0	0
Dim=150	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64765	65240	100,000	100,000	100,000	100,000
		Std	376.4970	200.6025	0	0	0	0
Dim=200	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64855	64840	100,000	100,000	100,000	100,000
		Std	438.3256	455.8811	0	0	0	0
Dim=300	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	65083	64980	100,000	100,000	100,000	100,000
		Std	306.0689	334.4063	0	0	0	0
Dim=400	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64158	64965	100,000	100,000	100,000	100,000
		Std	589.7228	306.0172	0	0	0	0
Dim=500	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	64758	6.5069e+04	100,000	100,000	100,000	100,000
		Std	312.6867	495.4100	0	0	0	0

Table 5

Function Evaluations for Multimodal functions

Dimension/ Function	Algo	FEs	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}
Dim=10	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	74575	100,000	6.1038e+04	100,000
		Std	0	0	431.8004	0	1.6360e+03	0
	TLBO	Mean	100,000	16200	1.2633e+04	100,000	20225	1.6912e+05
		Std	0	2.8036e+03	2.7029e+03	0	3.0704e+03	7.6285e+05
Dim=20	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	89300	100,000	100,000	100,000
		Std	0	0	1.8816e+03	0	0	0
	TLBO	Mean	100,000	15540	9160	100,000	16925	1.7867e+04
		Std	0	2.0651e+03	483.9493	0	4.3732e+03	818.0261
Dim=30	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	15369	9200	100,000	1.9727e+04	3.3204e+05
		Std	0	2.0516e+03	9.2968e+03	0	2.0449e+03	1.8693e+06
Dim=50	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	11000	8875	100,000	13100	7050
		Std	0	898.5703	315.6626	0	1.8865e+003	263.4930
Dim=100	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	10090	9200	100,000	12550	3900
		Std	0	790.1159	165.8312	0	1.2679e+03	64.1689
Dim=150	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	9500	8875	100,000	10425	2620
		Std	0	559.1529	193.9021	0	788.0282	162.3359
Dim=200	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	9000	9200	100,000	10025	2280
		Std	0	415.1161	0	0	470.6266	99.6546
Dim=300	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	8700	8875	100,000	9325	1860
		Std	0	300.1191	210.1912	0	832.2357	137.9655
Dim=400	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	8600	8875	100,000	8950	1.4333e+03
		Std	0	250.1106	115.1919	0	253.5463	47.8091
Dim=500	PSO	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	DE	Mean	100,000	100,000	100,000	100,000	100,000	100,000
		Std	0	0	0	0	0	0
	TLBO	Mean	100,000	8600	8875	100,000	8966	1.3656e+03
		Std	0	219.7177	190.1612	0	383.5706	48.2559

The fitness values and the number of function evaluations for six unmodal functions are shown in Table 2 and Table 4 respectively. Similarly, for multimodal functions it is shown in Table 3 and Table 5. The results are shown after 30 independent runs. The mean and standard deviations are calculated for obtained global minimum values and for the number of function evaluations in each algorithm with different dimensions.

Discussion of Results

In this work, all functions are run for 10^5 function evaluations (FFs) and the simulation is terminated when it reached the maximum number of evaluations or when it reached the global minima value for each test function.

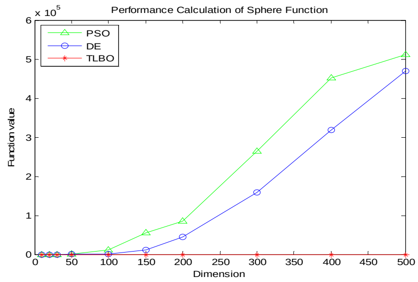

Fig. 1. Sphere

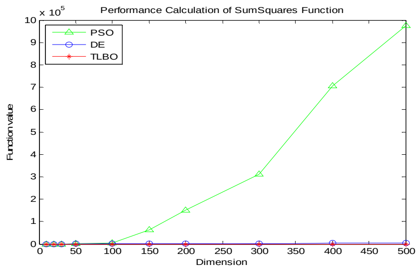

Fig. 2. SumSquares

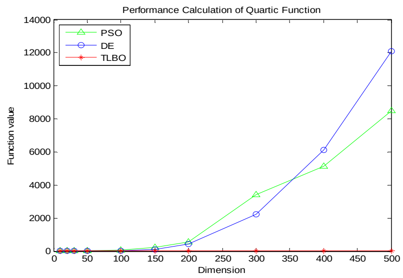

Fig. 3. Quartic

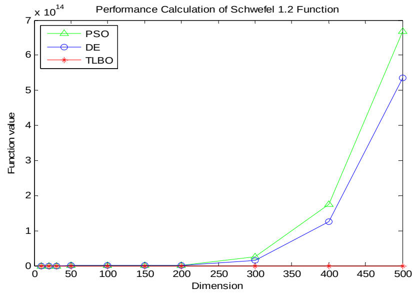

Fig. 4. Zakharov

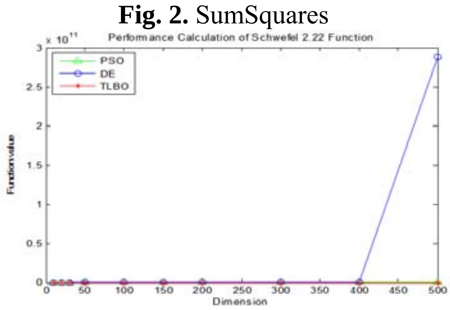

Fig. 5. Schwefel 1.2

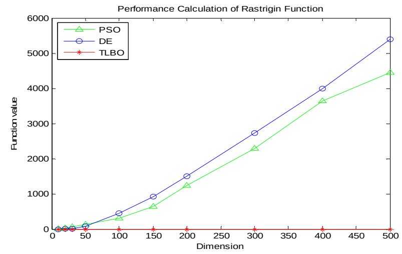

Fig. 6. Schwefel 2.22

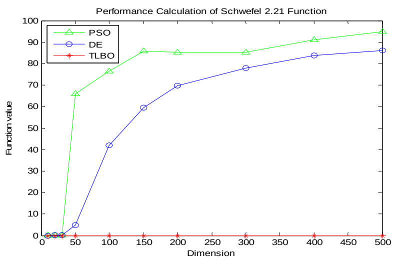

Fig. 7. Schwefel 2.21

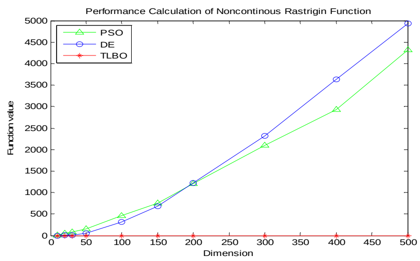

Fig. 8. Rastrigin

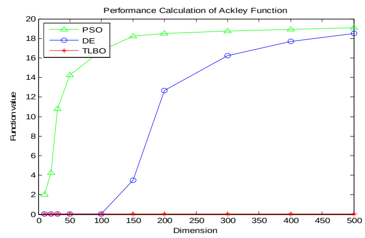

Fig. 9. Griewank

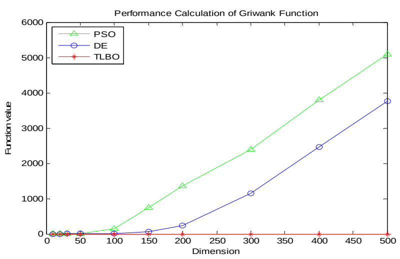

Fig. 10. Ackley

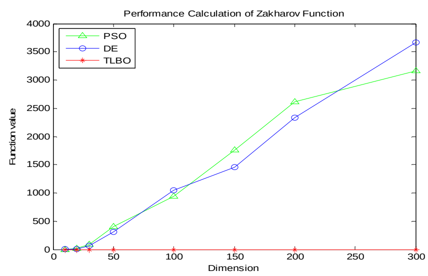

Fig. 11. Noncontinuous Rastrigin

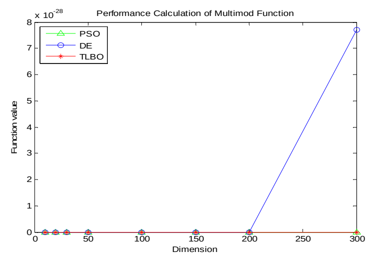

Fig. 12. Multimod

From the Table 2 and 3 it is clear that, as the dimension increases, it is difficult for both PSO and DE to locate the global best position and the algorithm traps into local point, but it is not the case for TLBO. In other words, increasing dimensions will not effect for searching global best position in TLBO. From the Table 4 it can be verified that the number of FEs are considerably less for Sphere and SumSquare functions in TLBO compared to other two algorithms. Rastrigin, Noncontinous Rastrigin, Griwank, Multimod functions are finding optimal global values in TLBO with less FEs compared to DE and PSO particularly in increasing dimensions as shown in Table 5. In general all the functions experimented in our work, TLBO outperforms other two approaches. Fig. 1 to Fig. 12 presents the fitness curve of all tested functions against all algorithms.

It is clearly evident from above figures that TLBO is able to locate global minimum values for all functions even in high dimensions. However, PSO and DE fail to obtain optimal values even upto 100000 function evaluations. But on the other hand TLBO is able to find best global minimum values for all dimensions up to 500 within 100000 FEs. This fact establishes the ability of TLBO to optimize high dimension functions over other two well know optimization techniques DE and PSO.

4. Conclusion and future research

In this paper, a new optimization technique called Teaching-Learning based Optimization (TLBO) is implemented for solving high dimensional real parameter optimization benchmark functions. The results are compared with that of other two very popular optimization techniques known as Differential Evolution (DE) and Particle Swarm Optimization (PSO). From the simulation results, it is clearly noted that TLBO is a very powerful optimization technique in handling high dimension functions. Twelve benchmark functions belonging two unimodal and multimodal category are simulated with different dimensions ranging up to 500. In all functions, TLBO could able to locate global minimum function values with less number of function evaluations (FEs) compared to DE and PSO. This has clearly demonstrated the capability of TLBO as candidate to solve very high dimension industrial application problems. As a further research, it can be seen how TLBO address the multi-objective function optimization problems.

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