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Optimal trade-credit policy for perishable items deeming imperfect production and stock dependent demand

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C H R O N I C L E

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A B S T R A C T

Trade credit is the most succeeding economic phenomenon which is used by the supplier for encouraging the retailers to buy more quantity. In this article, a mathematical model with stock dependent demand and deterioration is developed to investigate the retailer's optimal inventory policy under the scheme of permissible delay in payment. It is assumed that defective items are produced during the production process and delay period is progressive. The objective is to minimize the total average cost of the system. To exemplify hypothesis of the proposed model numerical examples and sensitivity analysis are provided. Finally, the convexities of the cost functions and the effects of changing parameters are represented through the graphs.

1. Introduction

In long-established inventory models, it is often assumed that the purchasing cost for the items is paid by the retailer to the supplier as soon as the items have been received. In practice, a delay period known as trade credit period is offered by the supplier to the retailer, in paying for purchasing cost. Up to the end of the trade credit of a cycle, the retailer is free of charge, but he/she is charged on an interest for those items not being sold before this end. During the trade credit period, the retailer can accumulate revenues by selling items and earning interests. Goyal (1985) is the first person who developed the EOQ model under conditions of permissible delay in payments. Shah et al. (1988) studied the same model, incorporating shortages. Later on, Aggarwal and Jaggi (1995) discussed the inventory model considering deterioration and permissible delay in payment. Other motivating mechanisms in this research area are those of Teng (2002), Ouyang et al. (2006), Khanra et al. (2011), Singh et al. (2011), Teng et al. (2012), Singhal & Singh (2013) and Singh and Sharma (2013).

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Deterioration of goods plays an important role in inventory system since in real life situations most of the physical goods deteriorate over time. Foods, pharmaceuticals, drugs, radioactive substances are some examples of items in which sufficient deterioration can take place during the normal cargo period and thus it plays an important role in analyzing the system. Generally, deterioration is defined as decay, damage or spoilage and obsolescence, which result in decrease of value of the original one. Ghare and Schrader (1963) presented the first model for decaying items. Covert and Philip (1973) extended their model considering Weibull distribution deterioration. Raafat (1991) presented a survey of literature on deteriorating inventory models. Hariga and Benkherouf (1994) proposed an inventory model for deteriorating items and later on Goyal and Giri (2001) provided a detailed review of deteriorating inventory literatures. Some other models dealing with the same issue are Yang and Wee (2006) and Kumar et al. (2012).

Many business practices reveal that the presence of a larger quantity of goods displayed attract customers to buy more quantity. This phenomenon implies that the demand may have a positive correlative with stock level. As Levin et al. (1972) observed that ‘‘large piles of consumer goods displayed in a supermarket will lead the customer to buy more. Yet, too much piled up in everyone's way leaves a negative impression on buyer and employee alike". Gupta and Vrat (1986) and Baker and Urban (1988) were the first to initiate a class of inventory models in which the demand rate is inventory dependent. Mandal and Phaujdar (1989) then developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Other papers related to this research area are by Zhou and Yang (2005), Lee and Dye (2012).

Most of the existing production inventory models ignored the presence of the imperfect production process. However, in real life situation, it is often observed that some of the items may be imperfect in nature, which are reworked at a cost to make them perfect. The production of defective items may be due to machine breakdown, labor problem, etc. Rosenblatt and Lee (1986) presumed that the time between the beginnings of the production run until the process goes out of control is exponential and that defective items can be reworked instantly at a cost and kept in stock. Kim and Hong (1999) determined the optimal production run length in deteriorating production processes.

Salameh and Jaber (2000) developed an economic production inventory model for items with imperfect quality items. Goyal and Barron (2002) extended the model presented by Salameh and Jaber's (2000). An inventory model is developed by Chung and Hou (2003) to obtain an optimal run time for a deteriorating production system with shortages. Yu et al. (2005) generalized the models of Salameh and Jaber (2000), considering deterioration and partial backordering. Later on, Kang (2010) presented an inventory model considering trade credit and items of imperfect quality. Recently, Sarkar and Moon (2011), Singh and Singh (2011), Sarkar (2012) and Singh et al. (2012) established some motivating inventory models with imperfect production processes.

An enormous work has been done in the field of trade-credit. Many previous economic order quantity inventory models are developed with trade-credit, a very few production inventory models are developed under allowable delay in payment. In addition, the inventory models for perishable items with imperfect production, stock dependent demand under trade-credit in which production rate depends on demand factor are much rare. Therefore, the present model is developed with these unique features. This model is an extension of the model Sarkar (2012) by considering deterioration and demand dependent production. The most favorable solution of the proposed model not only exists but also is unique. To obtain the optimal solution some lemmas are provided and with the help of sensitivity analysis, the effect of change in the parameters on the optimal policy is also disclosed.

2. Assumptions and notations

The following assumptions and notations are taken to discuss the model.

Assumptions

1. The inventory organism deals with a single type of items.
2. The replenishment rate is finite.
3. The delay in payment is offered to the retailer.
4. The demand is stock-dependent.
5. There is no repair or replacement of the deteriorated units.
6. Shortages are not permitted.
7. The lead time is zero.
8. The production of imperfect items is considered.

Notations

$I_1(t)$	On-hand inventory at time t where $0 \le t \le t_1$ (units)
$I_2(t)$	On-hand inventory at time t where $t_1 \le t \le T$ (units)
p	Selling price per unit ($/units)
D	Stock-dependent demand i.e. $D = a + mI(t)$, $a > 0$; $m > 0$ (units)
T	Length of inventory cycle (year)
P	Production rate (units per year), defined as P = ka, and k>1
R	The 1st offered trade-credit period without any charge (years)
S	The 2nd offered trade-credit period with charge (years)
I_e	Rate of interest earned due to financing inventory (/year)
I_{c1}	Rate of interest charged due to the credit balance for [R, S] ($/year)
I_{c2}	Rate of interest charged due to the credit balance for [S, T] ($/year)
C_A	Ordering cost per order ($/order)
C	Production cost ($/unit)
C_d	Deterioration cost ($/unit)
C_h	Holding cost ($/unit item/unit time)
C_p	Purchasing cost ($/unit)
C_r	Rework cost for the defective cost ($/item)
Z_i	Total cost of the system for i={1,2,3} ($)

3. Formulation of the Model

We consider an inventory model with stock-dependent demand model with different types of delay period. Depending on this policy, there may arise some cases:

Case (1): If the retailer pays the purchasing cost within the time R (i.e., $T \le R$), then there is no interest charged.

Case (2): If the retailer pays the purchasing cost after R and before S (i.e., $R \le T \le S$), then the supplier can charge a rate of interest I_{c1} to the retailer.

Case (3): If the retailer pays the purchasing cost after S and before T (i.e., T ≥ S), the supplier can charge a rate of interest I_{c2} on the unpaid balance (see Figs. 1–3).

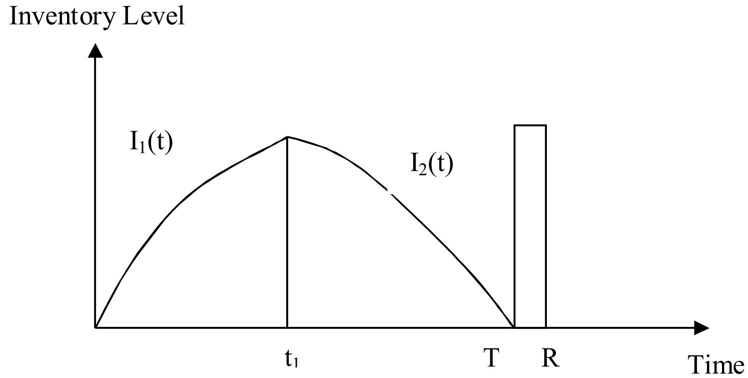

Fig. 1. Inventory versus time (Case-1: $T \le R$)

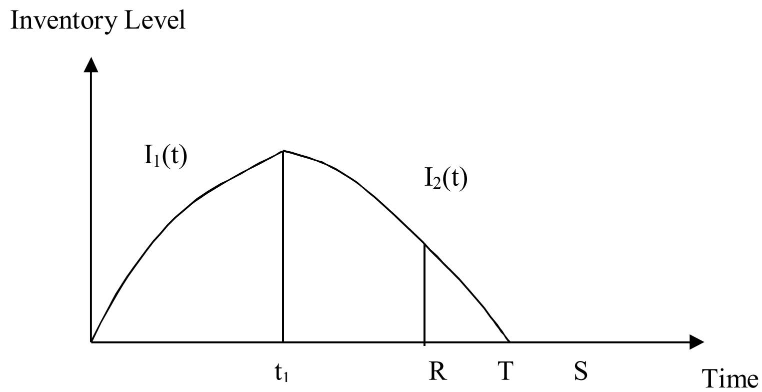

Fig. 2. Inventory versus time (Case-2: $R \le T \le S$)

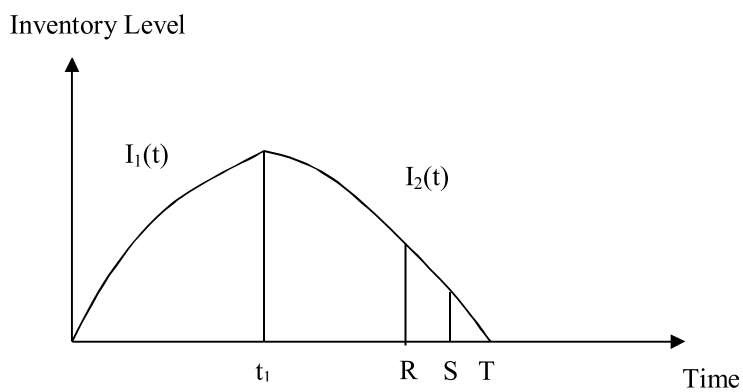

Fig. 3. Inventory versus time (Case-3: $T \ge S$)

Now, the present state of the on-hand inventory is described by the following differential equations:

$$I_1'(t) = P - D(I_1(t)) - \theta I_1(t) = ka - \left(a + mI_1(t)\right) - \theta I_1(t), \quad 0 \le t \le t_1 \tag{1}$$

and

$$I_2'(t) = -D(I_2(t)) - \theta I_2(t) = -\left(a + mI_2(t)\right) - \theta I_2(t), \qquad t_1 \le t \le T \tag{2}$$

with boundary conditions $I_1(0) = 0$ *and* $I_2(T) = 0$.

The solutions of the equations (1) and (2) are given as follows:

$$I_1(t) = \frac{(k-1)a}{(m+\theta)}\left[1 - e^{-(m+\theta)t}\right], \qquad 0 \le t \le t_1 \tag{3}$$

and

$$I_2(t) = \frac{a}{(m+\theta)}\left[e^{-(m+\theta)(t-T)} - 1\right], \qquad t_1 \le t \le T \tag{4}$$

Since $I_1(t_1) = I_2(t_1)$, we have

$$\frac{(k-1)a}{(m+\theta)}\left[1 - e^{-(m+\theta)t_1}\right] = \frac{a}{(m+\theta)}\left[e^{-(m+\theta)(t_1-T)} - 1\right] \Rightarrow t_1 = \frac{1}{(m+\theta)}\ln\left[1 - \frac{\left(1 - e^{(m+\theta)T}\right)}{k}\right]. \tag{5}$$

Now, different costs of the inventory system are as follows:

Ordering cost is OC and is given by

$$OC = \frac{C_A}{T}. \tag{6}$$

Inventory holding cost is HC and is given by

$$HC = \frac{C_h}{T}\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right] \Rightarrow HC = \frac{C_h}{T}\left[\frac{a}{(m+\theta)^2}\left\{(k-1)e^{-(m+\theta)t_1} + e^{-(m+\theta)(t_1-T)} - k\right\} + \frac{a(kt_1 - T)}{(m+\theta)}\right]. \tag{7}$$

Deterioration cost for deteriorating items is DC and is given by

$$DC = \frac{C_d}{T}\left[\int_0^{t_1} \theta I_1(t)dt + \int_{t_1}^{T} \theta I_2(t)dt\right] \Rightarrow DC = \frac{C_d\theta}{T}\left[\frac{a}{(m+\theta)^2}\left\{(k-1)e^{-(m+\theta)t_1} + e^{-(m+\theta)(t_1-T)} - k\right\} + \frac{a(kt_1 - T)}{(m+\theta)}\right]. \tag{8}$$

Production cost is PRC and is given by

$$PRC = \frac{C}{T}\int_0^{t_1} Pdt = \frac{Ckat_1}{T}. \tag{9}$$

Purchasing cost is PUC and is given by

$$PUC = \frac{C_p}{T}\int_0^{t_1} Pdt = \frac{C_p kat_1}{T}. \tag{10}$$

Along with the trade credit, the paper considers the production of imperfect items. The lifetime of defective item follows a Weibull distribution defined as $\psi(t) = \alpha t^{\beta}$, $\beta > -1$, where α, β are two parameters and t is the time to failure. Hence, the total number of defective items is:

$$N = \int_0^{t_1} P\psi(t)e^{\left[-\int_0^{t}\psi(\tau)d\tau\right]}dt = ka\left(1 - e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}\right). \tag{11}$$

The rework cost is RC and is given by

$$RC = \frac{C_r N}{T} = \frac{C_r ka}{T}\left(1 - e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}\right). \tag{12}$$

Now, for different delay periods:

Case (1): $T \le R$

In this case, interest earned is IE_1 and is given by

$$IE_1 = \frac{pI_e}{T}\left[\int_0^{t_1}(t_1 - t)D\left(I_1(t)\right)dt + \int_{t_1}^{T}(T - t)D\left(I_2(t)\right)dt + (R - T)\left\{\int_0^{t_1}D\left(I_1(t)\right)dt + \int_{t_1}^{T}D\left(I_2(t)\right)dt\right\}\right].$$

$$IE_1=\frac{pI_e}{T}\Bigg[(R-T+t_1)\Bigg\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\Big(e^{-(m+\theta)t_1}-1\Big)\Bigg\}+R\Bigg\{a(T-t_1)+\frac{ma}{(m+\theta)^2}\Big(e^{-(m+\theta)(t_1-T)}-1\Big)$$
$$-\frac{ma(T-t_1)}{(m+\theta)}\Bigg\}-\Bigg\{\frac{at_1^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\Big(e^{-(m+\theta)t_1}-1\Big)\Bigg\}$$
$$-\Bigg\{\frac{a(T^2-t_1^2)}{2}+\frac{ma}{(m+\theta)^2}\Big(t_1e^{-(m+\theta)(t_1-T)}-T\Big)+\frac{ma}{(m+\theta)^3}\Big(e^{-(m+\theta)(t_1-T)}-1\Big)-\frac{ma(T^2-t_1^2)}{2(m+\theta)}\Bigg\}\Bigg]. \tag{13}$$

In this case, interest charged is IC_1 and is given by

$$IC_1=0. \tag{14}$$

Case (2): R ≤ T ≤ S

In this case, interest earned is IE_2 and is given by

$$IE_2=\frac{pI_e}{T}\left[\int_0^{t_1}(t_1-t)D(I_1(t))dt+\int_{t_1}^{T}(T-t)D(I_2(t))dt\right].$$
$$IE_2=\frac{pI_e}{T}\Bigg[t_1\Bigg\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\Big(e^{-(m+\theta)t_1}-1\Big)\Bigg\}+T\Bigg\{a(T-t_1)+\frac{ma}{(m+\theta)^2}\Big(e^{-(m+\theta)(t_1-T)}-1\Big)-\frac{ma(T-t_1)}{(m+\theta)}\Bigg\}$$
$$-\Bigg\{\frac{at_1^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\Big(e^{-(m+\theta)t_1}-1\Big)\Bigg\}$$
$$-\Bigg\{\frac{a(T^2-t_1^2)}{2}+\frac{ma}{(m+\theta)^2}\Big(t_1e^{-(m+\theta)(t_1-T)}-T\Big)+\frac{ma}{(m+\theta)^3}\Big(e^{-(m+\theta)(t_1-T)}-1\Big)-\frac{ma(T^2-t_1^2)}{2(m+\theta)}\Bigg\}\Bigg]. \tag{15}$$

In this case, interest charged is IC_2 and is given by

$$IC_2=\frac{I_{c1}C_P}{T}\int_R^T I_2(t)dt=\frac{I_{c1}C_Pa}{T(m+\theta)}\left[R-T-\frac{1}{(m+\theta)}+\frac{e^{-(m+\theta)(R-T)}}{(m+\theta)}\right]. \tag{16}$$

Case (3): $T\geq S$

In this case, interest earned is IE_3 and is given by

$$IE_3=\frac{pI_e}{T}\left[\int_0^{t_1}(t_1-t)D(I_1(t))dt+\int_{t_1}^{S}(T-t)D(I_2(t))dt\right].$$
$$IE_3=\frac{pI_e}{T}\Bigg[t_1\Bigg\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\Big(e^{-(m+\theta)t_1}-1\Big)\Bigg\}+T\Bigg\{a(S-t_1)+\frac{ma}{(m+\theta)^2}\Big(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\Big)$$
$$-\frac{ma(S-t_1)}{(m+\theta)}\Bigg\}-\Bigg\{\frac{at_1^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\Big(e^{-(m+\theta)t_1}-1\Big)\Bigg\}$$
$$-\Bigg\{\frac{a(S^2-t_1^2)}{2}+\frac{ma}{(m+\theta)^2}\Big(t_1e^{-(m+\theta)(t_1-T)}-Se^{-(m+\theta)(S-T)}\Big)+\frac{ma}{(m+\theta)^3}\Big(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\Big)-\frac{ma(S^2-t_1^2)}{2(m+\theta)}\Bigg\}\Bigg]. \tag{17}$$

In this case, interest charged is IC_3 and is given by

$$IC_3=\frac{I_{c2}C_P}{T}\int_S^T I_2(t)dt=\frac{I_{c2}C_Pa}{T(m+\theta)}\left[S-T-\frac{1}{(m+\theta)}+\frac{e^{-(m+\theta)(S-T)}}{(m+\theta)}\right]. \tag{18}$$

Thus, the total average cost for case (1): is $Z_1(T)$ and is given by,

$$Z_1(T)=OC+HC+DC+PRC+PUC+RC+IC_1-IE_1. \tag{19}$$

The total average cost for case (2) is $Z_2(T)$ and is given by

$$Z_2(T) = OC + HC + DC + PRC + PUC + RC + IC_2 - IE_2 \quad . \tag{20}$$

The total average cost for case (3) is $Z_3(T)$ and is given by

$$Z_3(T) = OC + HC + DC + PRC + PUC + RC + IC_3 - IE_3 \quad . \tag{21}$$

Our objective is to minimize the total cost of the inventory system. The necessary conditions for the existence of the optimal solutions are

$$\frac{dZ_1(T)}{dT} = 0, \tag{22}$$

$$\frac{dZ_2(T)}{dT} = 0, \tag{23}$$

$$\frac{dZ_3(T)}{dT} = 0. \tag{24}$$

Using the software Mathematica-8.0, from eq. (22) to Eq. (24) we can determine the optimum values of $T = T_i^*$, where $i=1, 2, 3$ and the optimal value $Z_i(T_i^*)$, where $i=1,2,3$ of the total cost can be determined by (21) provided they satisfy the sufficiency conditions for minimizing $Z_i(T_i^*)$, where $i=1, 2, 3$ given by

$$\left.\frac{d^2 Z_i(T)}{dT^2}\right|_{T=T_i*} > 0, \; where \; i = 1,2,3$$

For the cost minimization we may formulate the three lemmas (motivated by Sarkar (2012)) as follows:

Lemma 1. $Z_1(T^*)$ must have a minimum value at T^* if it satisfies the equation $T^* = \frac{\left[C_A + \phi_2\left(C_p + C\right) + \phi_4 C_r ka + \phi_1\left(C_h + \theta C_d\right) - \phi_3 pI_e\right]}{\left[\phi_5\left(C_h + \theta C_d\right) + \phi_{11}\left(C_p + C\right) + \phi_{10} C_r ka - \phi_6 pI_e\right]}$ and the inequality $\phi_{16}\left(T^*\right)^2 + \phi_{17}T^* + \phi_{18} > 0$ where all the values of ϕ_i's are given in Appendix 1.

Proof. For minimization of the total cost Z_1, $\left.\frac{dZ_1}{dT}\right|_{T=T^*} = 0 \; and \; \left.\frac{d^2 Z_1}{dT^2}\right|_{T=T^*} > 0$ must be satisfied.

Now,

$$\begin{aligned}
Z_1 = {} & \frac{C_A}{T} + \frac{\left(C_h + \theta C_d\right)}{T}\left\{\frac{a}{(m+\theta)^2}\left\{(k-1)e^{-(m+\theta)t_1} + e^{-(m+\theta)(t_1-T)} - k\right\} + \frac{a\left(kt_1 - T\right)}{(m+\theta)}\right\} + \frac{\left(C_p + C\right)kat_1}{T} \\
& - \frac{pI_e}{T}\left[\left(R - T + t_1\right)\left\{at_1 + \frac{m(k-1)at_1}{(m+\theta)} + \frac{m(k-1)a}{(m+\theta)^2}\left(e^{-(m+\theta)t_1} - 1\right)\right\} + R\left\{a\left(T - t_1\right) + \frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)} - 1\right)\right.\right. \\
& \left. - \frac{ma\left(T - t_1\right)}{(m+\theta)}\right\} - \left\{\frac{at_1^2}{2} + \frac{m(k-1)at_1^2}{2(m+\theta)} + \frac{m(k-1)at_1 e^{-(m+\theta)t_1}}{(m+\theta)^2} + \frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1} - 1\right)\right\} \\
& \left. - \left\{\frac{a\left(T^2 - t_1^2\right)}{2} + \frac{ma}{(m+\theta)^2}\left(t_1 e^{-(m+\theta)(t_1-T)} - T\right) + \frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)} - 1\right) - \frac{ma\left(T^2 - t_1^2\right)}{2(m+\theta)}\right\}\right] + \frac{C_r ka}{T}\left(1 - e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}\right).
\end{aligned}$$

Differentiating the above expression with respect to T, we get

$$\frac{dZ_1}{dT} = \frac{\phi_5\left(C_h + \theta C_d\right)}{T} - \frac{\phi_1\left(C_h + \theta C_d\right)}{T^2} - \frac{C_A}{T^2} - \frac{\phi_2\left(C_p + C\right)}{T^2} + \frac{\phi_{11}\left(C_p + C\right)}{T} + \frac{\phi_3 pI_e}{T^2} - \frac{\phi_6 pI_e}{T} - \frac{\phi_4 C_r ka}{T^2} + \frac{\phi_{10} C_r ka}{T},$$

and again, differentiating the above expression with respect to T, we find

$$\begin{aligned}
\frac{d^2 Z_1}{dT^2} = {} & \frac{2C_A}{T^3} + \frac{2\phi_1\left(C_h + \theta C_d\right)}{T^3} - \frac{2\phi_5\left(C_h + \theta C_d\right)}{T^2} + \frac{\phi_{12}\left(C_h + \theta C_d\right)}{T} + \frac{2\phi_2\left(C_p + C\right)}{T^3} - \frac{2\phi_{11}\left(C_p + C\right)}{T^2} \\
& + \frac{\phi_{13}\left(C_p + C\right)}{T} - \frac{2\phi_3 pI_e}{T^3} + \frac{2\phi_6 pI_e}{T^2} - \frac{\phi_{14} pI_e}{T} + \frac{2\phi_4 C_r ka}{T^3} - \frac{2\phi_{10} C_r ka}{T^2} + \frac{\phi_{15} C_r ka}{T},
\end{aligned}$$

For minimum of Z_1, $\left.\frac{dZ_1}{dT}\right|_{T=T^*}=0 \; and \; \left.\frac{d^2Z_1}{dT^2}\right|_{T=T^*}>0$ imply that

$$\left[\frac{\phi_5\left(C_h+\theta C_d\right)}{T}+\frac{\phi_{11}\left(C_p+C\right)}{T}+\frac{\phi_3 pI_e}{T^2}+\frac{\phi_{10}C_r ka}{T}\right]_{T=T^*}=\left[\frac{C_A}{T^2}+\frac{\phi_1\left(C_h+\theta C_d\right)}{T^2}+\frac{\phi_2\left(C_p+C\right)}{T^2}+\frac{\phi_6 pI_e}{T}+\frac{\phi_4 C_r ka}{T^2}\right]_{T=T^*},$$

and

$$\left[\frac{2C_A}{T^3}+\frac{2\phi_1\left(C_h+\theta C_d\right)}{T^3}+\frac{\phi_{12}\left(C_h+\theta C_d\right)}{T}+\frac{2\phi_2\left(C_p+C\right)}{T^3}+\frac{\phi_{13}\left(C_p+C\right)}{T}+\frac{2\phi_6 pI_e}{T^2}+\frac{2\phi_4 C_r ka}{T^3}+\frac{\phi_{15}C_r ka}{T}\right]_{T=T^*}$$

$$>\left[\frac{2\phi_5\left(C_h+\theta C_d\right)}{T^2}+\frac{2\phi_{11}\left(C_p+C\right)}{T^2}+\frac{2\phi_3 pI_e}{T^3}+\frac{\phi_{14}pI_e}{T}+\frac{2\phi_{10}C_r ka}{T^2}\right]_{T=T^*}.$$

After some simplification, we get

$$T^*=\frac{\left[C_A+\phi_2\left(C_p+C\right)+\phi_4 C_r ka+\phi_1\left(C_h+\theta C_d\right)-\phi_3 pI_e\right]}{\left[\phi_5\left(C_h+\theta C_d\right)+\phi_{11}\left(C_p+C\right)+\phi_{10}C_r ka-\phi_6 pI_e\right]} \quad \text{and} \quad \phi_{16}\left(T^*\right)^2+\phi_{17}T^*+\phi_{18}>0.$$

Hence the proof.

Lemma 2. $Z_2(T^*)$ must have a minimum value at T^* if it satisfies the equation $T^*=\frac{\left[C_A+\phi_1\left(C_h+\theta C_d\right)+\phi_2\left(C_p+C\right)+\phi_4 C_r ka+\phi_{21}C_p I_{c1}-\phi_{19}pI_e\right]}{\left[\phi_5\left(C_h+\theta C_d\right)+\phi_{11}\left(C_p+C\right)+\phi_{10}C_r ka+\phi_{22}C_p I_{c1}-\phi_{20}pI_e\right]}$ and the inequality $\phi_{25}\left(T^*\right)^2+\phi_{26}T^*+\phi_{27}>0$ where all the values of ϕ_i's are given in Appendix 2.

Proof. For minimization of the total cost Z_2, $\left.\frac{dZ_2}{dT}\right|_{T=T^*}=0 \; and \; \left.\frac{d^2Z_2}{dT^2}\right|_{T=T^*}>0$ must be satisfied.

Now,

$$Z_2=\frac{C_A}{T}+\frac{\left(C_h+\theta C_d\right)}{T}\left\{\frac{a}{\left(m+\theta\right)^2}\left\{\left(k-1\right)e^{-(m+\theta)t_1}+e^{-(m+\theta)(t_1-T)}-k\right\}+\frac{a\left(kt_1-T\right)}{\left(m+\theta\right)}\right\}+\frac{\left(C_p+C\right)kat_1}{T}$$

$$-\frac{pI_e}{T}\left[t_1\left\{at_1+\frac{m\left(k-1\right)at_1}{\left(m+\theta\right)}+\frac{m\left(k-1\right)a}{\left(m+\theta\right)^2}\left(e^{-(m+\theta)t_1}-1\right)\right\}+T\left\{a\left(T-t_1\right)+\frac{ma}{\left(m+\theta\right)^2}\left(e^{-(m+\theta)(t_1-T)}-1\right)\right.\right.$$

$$\left.-\frac{ma\left(T-t_1\right)}{\left(m+\theta\right)}\right\}-\left\{\frac{at_1^2}{2}+\frac{m\left(k-1\right)at_1^2}{2\left(m+\theta\right)}+\frac{m\left(k-1\right)at_1e^{-(m+\theta)t_1}}{\left(m+\theta\right)^2}+\frac{m\left(k-1\right)a}{\left(m+\theta\right)^3}\left(e^{-(m+\theta)t_1}-1\right)\right\}$$

$$\left.-\left\{\frac{a\left(T^2-t_1^2\right)}{2}+\frac{ma}{\left(m+\theta\right)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-T\right)+\frac{ma}{\left(m+\theta\right)^3}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma\left(T^2-t_1^2\right)}{2\left(m+\theta\right)}\right\}\right]$$

$$+\frac{C_r ka}{T}\left(1-e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}\right)+\frac{I_{c1}C_p a}{T\left(m+\theta\right)}\left\{R-T-\frac{1}{\left(m+\theta\right)}+\frac{e^{-(m+\theta)(R-T)}}{\left(m+\theta\right)}\right\}.$$

Differentiating the above expression with respect to T, we get

$$\frac{dZ_2}{dT}=\frac{\phi_5\left(C_h+\theta C_d\right)}{T}-\frac{\phi_1\left(C_h+\theta C_d\right)}{T^2}-\frac{C_A}{T^2}-\frac{\phi_2\left(C_p+C\right)}{T^2}+\frac{\phi_{11}\left(C_p+C\right)}{T}+\frac{\phi_{19}pI_e}{T^2}-\frac{\phi_{20}pI_e}{T}-\frac{\phi_4 C_r ka}{T^2}+\frac{\phi_{10}C_r ka}{T}-\frac{\phi_{21}C_p I_{c1}}{T^2}+\frac{\phi_{22}C_p I_{c1}}{T},$$

and again, differentiating the above expression with respect to T, we have

$$\frac{d^2Z_2}{dT^2}=\frac{2C_A}{T^3}+\frac{2\phi_1\left(C_h+\theta C_d\right)}{T^3}-\frac{2\phi_5\left(C_h+\theta C_d\right)}{T^2}+\frac{\phi_{12}\left(C_h+\theta C_d\right)}{T}+\frac{2\phi_2\left(C_p+C\right)}{T^3}-\frac{2\phi_{11}\left(C_p+C\right)}{T^2}+\frac{\phi_{13}\left(C_p+C\right)}{T}$$

$$-\frac{2\phi_{19}pI_e}{T^3}+\frac{2\phi_{20}pI_e}{T^2}-\frac{\phi_{23}pI_e}{T}+\frac{2\phi_4 C_r ka}{T^3}-\frac{2\phi_{10}C_r ka}{T^2}+\frac{\phi_{15}C_r ka}{T}+\frac{2\phi_{21}C_p I_{c1}}{T^3}-\frac{2\phi_{22}C_p I_{c1}}{T^2}+\frac{\phi_{24}C_p I_{c1}}{T}.$$

For minimum of Z_2, $\left.\frac{dZ_2}{dT}\right|_{T=T^*} = 0$ and $\left.\frac{d^2 Z_2}{dT^2}\right|_{T=T^*} > 0$ imply that

$$\left[\frac{\phi_5(C_h+\theta C_d)}{T}+\frac{\phi_{11}(C_p+C)}{T}+\frac{\phi_{19} pI_e}{T^2}+\frac{\phi_{10} C_r ka}{T}+\frac{\phi_{22} C_p I_{c1}}{T}\right]_{T=T^*}=\left[\frac{C_A}{T^2}+\frac{\phi_1(C_h+\theta C_d)}{T^2}+\frac{\phi_2(C_p+C)}{T^2}+\frac{\phi_{20} pI_e}{T}+\frac{\phi_4 C_r ka}{T^2}+\frac{\phi_{21} C_p I_{c1}}{T^2}\right]_{T=T^*},$$

and

$$\left[\frac{2C_A}{T^3}+\frac{2\phi_1(C_h+\theta C_d)}{T^3}+\frac{\phi_{12}(C_h+\theta C_d)}{T}+\frac{2\phi_2(C_p+C)}{T^3}+\frac{\phi_{13}(C_p+C)}{T}+\frac{2\phi_{20} pI_e}{T^2}+\frac{2\phi_4 C_r ka}{T^3}+\frac{\phi_{15} C_r ka}{T}+\frac{2\phi_{21} C_p I_{c1}}{T^3}+\frac{\phi_{24} C_p I_{c1}}{T}\right]_{T=T^*}$$

$$>\left[\frac{2\phi_5(C_h+\theta C_d)}{T^2}+\frac{2\phi_{11}(C_p+C)}{T^2}+\frac{2\phi_{19} pI_e}{T^3}+\frac{\phi_{23} pI_e}{T}+\frac{2\phi_{10} C_r ka}{T^2}+\frac{2\phi_{22} C_p I_{c1}}{T^2}\right]_{T=T^*}.$$

After some simplification, we get

$$T^*=\frac{\left[C_A+\phi_1(C_h+\theta C_d)+\phi_2(C_p+C)+\phi_4 C_r ka+\phi_{21} C_p I_{c1}-\phi_{19} pI_e\right]}{\left[\phi_5(C_h+\theta C_d)+\phi_{11}(C_p+C)+\phi_{10} C_r ka+\phi_{22} C_p I_{c1}-\phi_{20} pI_e\right]} \quad and \quad \phi_{25}\left(T^*\right)^2+\phi_{26}T^*+\phi_{27}>0$$

Hence the proof.

Lemma 3. $Z_3(T^*)$ must have a minimum value at T^* if it satisfies the equation $T^*=\frac{\left[C_A+\phi_1(C_h+\theta C_d)+\phi_2(C_p+C)+\phi_4 C_r ka+\phi_{30} C_p I_{c2}-\phi_{28} pI_e\right]}{\left[\phi_5(C_h+\theta C_d)+\phi_{11}(C_p+C)+\phi_{10} C_r ka+\phi_{31} C_p I_{c2}-\phi_{29} pI_e\right]}$ and the inequality $\phi_{36}\left(T^*\right)^2+\phi_{37}T^*+\phi_{38}>0$ where all the values of ϕ_i's are given in Appendix 3.

Proof. For minimization of the total cost Z_3, $\left.\frac{dZ_3}{dT}\right|_{T=T^*}=0$ and $\left.\frac{d^2Z_3}{dT^2}\right|_{T=T^*}>0$ must be satisfied.

Now,

$$Z_3=\frac{C_A}{T}+\frac{(C_h+\theta C_d)}{T}\left\{\frac{a}{(m+\theta)^2}\left\{(k-1)e^{-(m+\theta)t_1}+e^{-(m+\theta)(t_1-T)}-k\right\}+\frac{a(kt_1-T)}{(m+\theta)}\right\}+\frac{(C_p+C)kat_1}{T}$$

$$-\frac{pI_e}{T}\left[t_1\left\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\left(e^{-(m+\theta)t_1}-1\right)\right\}+T\left\{a(S-t_1)+\frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\right)\right.\right.$$

$$\left.-\frac{ma(S-t_1)}{(m+\theta)}\right\}-\left\{\frac{at_1^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1}-1\right)\right\}-\left\{\frac{a(S^2-t_1^2)}{2}\right.$$

$$\left.\left.+\frac{ma}{(m+\theta)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-Se^{-(m+\theta)(S-T)}\right)+\frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\right)-\frac{ma(S^2-t_1^2)}{2(m+\theta)}\right\}\right]$$

$$+\frac{C_r ka}{T}\left(1-e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}\right)+\frac{I_{c2}C_P a}{T(m+\theta)}\left\{S-T-\frac{1}{(m+\theta)}+\frac{e^{-(m+\theta)(S-T)}}{(m+\theta)}\right\}.$$

Differentiating the above expression with respect to T, we get

$$\frac{dZ_3}{dT}=\frac{\phi_5(C_h+\theta C_d)}{T}-\frac{\phi_1(C_h+\theta C_d)}{T^2}-\frac{C_A}{T^2}-\frac{\phi_2(C_p+C)}{T^2}+\frac{\phi_{11}(C_p+C)}{T}+\frac{\phi_{28} pI_e}{T^2}-\frac{\phi_{29} pI_e}{T}-\frac{\phi_4 C_r ka}{T^2}$$

$$+\frac{\phi_{10} C_r ka}{T}-\frac{\phi_{30} C_p I_{c2}}{T^2}+\frac{\phi_{31} C_p I_{c2}}{T}.$$

and again, differentiating the above expression with respect to T, we have

$$\frac{d^2Z_3}{dT^2}=\frac{2C_A}{T^3}+\frac{2\phi_1(C_h+\theta C_d)}{T^3}-\frac{2\phi_5(C_h+\theta C_d)}{T^2}+\frac{\phi_{12}(C_h+\theta C_d)}{T}+\frac{2\phi_2(C_p+C)}{T^3}-\frac{2\phi_{11}(C_p+C)}{T^2}+\frac{\phi_{13}(C_p+C)}{T}$$

$$-\frac{2\phi_{28} pI_e}{T^3}+\frac{2\phi_{29} pI_e}{T^2}-\frac{\phi_{34} pI_e}{T}+\frac{2\phi_4 C_r ka}{T^3}-\frac{2\phi_{10} C_r ka}{T^2}+\frac{\phi_{15} C_r ka}{T}+\frac{2\phi_{30} C_p I_{c2}}{T^3}-\frac{2\phi_{31} C_p I_{c1}}{T^2}+\frac{\phi_{35} C_p I_{c2}}{T}$$

For minimum of Z_3, $\left.\frac{dZ_3}{dT}\right|_{T=T^*} = 0$ *and* $\left.\frac{d^2Z_3}{dT^2}\right|_{T=T^*} > 0$ imply that

$$\left[\frac{\phi_5(C_h+\theta C_d)}{T}+\frac{\phi_{11}(C_p+C)}{T}+\frac{\phi_{28}pI_e}{T^2}+\frac{\phi_{10}C_r ka}{T}+\frac{\phi_{31}C_pI_{c2}}{T}\right]_{T=T^*}=\left[\frac{C_A}{T^2}+\frac{\phi_1(C_h+\theta C_d)}{T^2}+\frac{\phi_2(C_p+C)}{T^2}+\frac{\phi_{29}pI_e}{T}+\frac{\phi_4C_rka}{T^2}+\frac{\phi_{30}C_pI_{c2}}{T^2}\right]_{T=T^*},$$

and

$$\left[\frac{2C_A}{T^3}+\frac{2\phi_1(C_h+\theta C_d)}{T^3}+\frac{\phi_{12}(C_h+\theta C_d)}{T}+\frac{2\phi_2(C_p+C)}{T^3}+\frac{\phi_{13}(C_p+C)}{T}+\frac{2\phi_{29}pI_e}{T^2}+\frac{2\phi_4C_rka}{T^3}+\frac{\phi_{15}C_rka}{T}+\frac{2\phi_{30}C_pI_{c2}}{T^3}+\frac{\phi_{35}C_pI_{c2}}{T}\right]_{T=T^*}$$
$$>\left[\frac{2\phi_5(C_h+\theta C_d)}{T^2}+\frac{2\phi_{11}(C_p+C)}{T^2}+\frac{2\phi_{28}pI_e}{T^3}+\frac{\phi_{34}pI_e}{T}+\frac{2\phi_{10}C_rka}{T^2}+\frac{2\phi_{31}C_pI_{c1}}{T^2}\right]_{T=T^*}.$$

After some simplification, we get

$$T^*=\frac{\left[C_A+\phi_1(C_h+\theta C_d)+\phi_2(C_p+C)+\phi_4C_rka+\phi_{30}C_pI_{c2}-\phi_{28}pI_e\right]}{\left[\phi_5(C_h+\theta C_d)+\phi_{11}(C_p+C)+\phi_{10}C_rka+\phi_{31}C_pI_{c2}-\phi_{29}pI_e\right]} \quad and \quad \phi_{36}(T^*)^2+\phi_{37}T^*+\phi_{38}>0.$$

Algorithm

Step 1: Determine T_1^* from equation (22), if $T_1^* \le R$ then evaluate $Z_1(T_1^*)$ from (19). Otherwise go to step 2.
Step 2: Determine T_2^* from equation (23), if $R \le T_1^* \le S$ then evaluate $Z_2(T_2^*)$ from (20). Otherwise go to step 3.
Step 3: Determine T_3^* from equation (24), if $T_1^* \ge S$ then evaluate $Z_3(T_3^*)$ from (21). Otherwise go to step 4.
Step 4: Find out TC = min{ $Z_1(T_1^*)$, $Z_2(T_2^*)$, $Z_3(T_3^*)$}.

4. Numerical Examples

All calculations are executed with the help of the software Mathematica 8.0, from where we get the optimal value. To illustrate the proposed model two examples are presented here in which Z_1 and Z_3 are the optimal solution.

Example 1. We consider the following parameter values on the basis of the previous study:

C_A=\$ 180/order, p = \$20/unit, a = 15, m = 0.5, k = 2, θ = 0.1, C = 2, C_d = 15/unit, R = 1.5 years, I_e = \$0.15/year, I_{c1} = \$0.18/year, I_{c2} = \$0.20/year, S = 1.74 years, C_h = \$14/unit/year, C_p = \$10/unit, α = 0.010, β = 0.053, C_r = \$1.5/item. Then the optimal solutions are:
In, case (1): {T_1^*=1.2529, $Z_1(T_1^*)$=398.759}, case (2): {T_2^*=1.7157, $Z_2(T_2^*)$=401.442}, case (3): {T_3^*=1.76178, $Z_3(T_3^*)$=400.971}. Among the above optimal solutions, the better optimal solution TC=min{ $Z_1(T_1^*)$, $Z_2(T_2^*)$, $Z_3(T_3^*)$}=398.759, T^*=1.2529. From the numerical example, Figs. 4–6 show the convexity of the cost function.

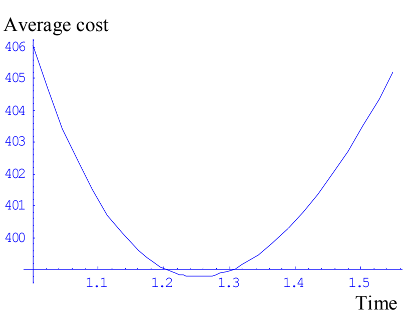

Fig. 4. Case 1: average cost versus cycle length

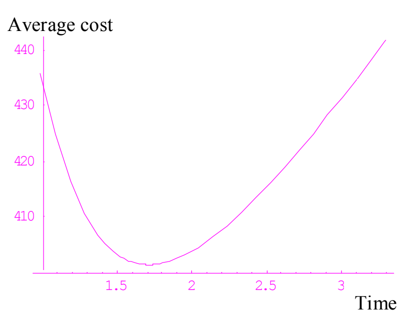

Fig. 5. Case 2: average cost versus cycle length

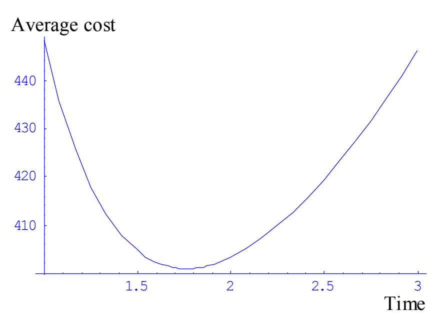

Fig. 6. Case 3: average cost versus cycle length

Example 2. We consider the following parameter values on the basis of the previous study: C_A=\$ 350/order, p = \$20/unit, a = 15, m = 0.5, k = 2, θ = 0.1, C = 2, C_d = 15/unit, R = 2.1 years, I_e = \$0.15/year, I_{c1} = \$0.18/year, I_{c2} = \$0.20/year, S = 2.75 years, C_h = \$14/unit/year, C_p = \$10/unit, α = 0.10, β = 0.53, C_r = \$1.5/item. Then the optimal solutions are:
For, case (1): $\{T_1^*=1.77939, Z_1(T_1^*)=480.611\}$, case (2): $\{T_2^*=2.66707, Z_2(T_2^*)=479.241\}$, case (3): $\{T_3^*=2.86137, Z_3(T_3^*)=476.712\}$. Among the above optimal solutions, the better optimal solution TC=min$\{Z_1(T_1^*), Z_2(T_2^*), Z_3(T_3^*)\}$=476.712, T^*=2.86137. From the numerical example, Figs. 7–9 show the convexity of the cost function.

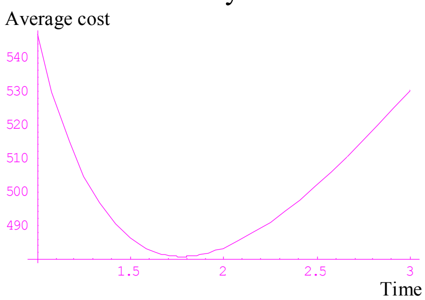

Fig. 7. Case 1: average cost versus cycle length

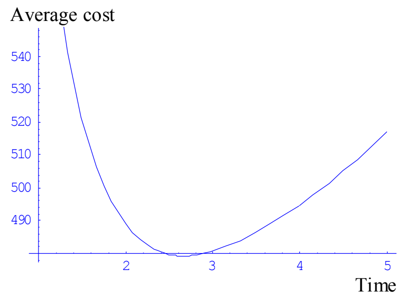

Fig. 8. Case 2: average cost versus cycle length

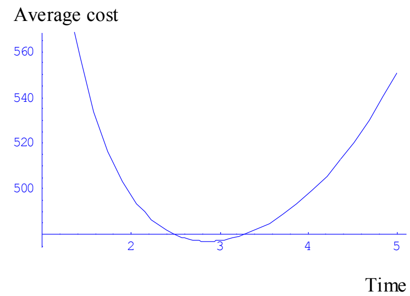

Fig. 9. Case 3: average cost versus cycle length

5. Sensitivity Analysis

Sensitivity analysis of the numerical example (1) and (2) are presented in Table 1 and 2 respectively as follows.

Table 1

(For example (1)) The effect of changing the parameter (*i*) while keeping all other parameters unchanged

Parameter (i)	% Change	T_1^*	$Z_1(T_1^*)$	T_2^*	$Z_2(T_2^*)$	T_3^*	$Z_3(T_3^*)$	Optimal Solution	TC
C_p	-20	1.27669	363.199	1.78904	363.798	1.80790	363.295	$Z_1(T_1^*)$	363.199
	-10	1.26464	380.991	1.75041	382.663	1.78406	382.157	$Z_1(T_1^*)$	380.991
	+10	1.24146	416.502	1.68424	420.145	1.74091	419.743	$Z_1(T_1^*)$	416.502
	+20	1.23031	434.221	1.65554	438.778	1.72130	438.474	$Z_1(T_1^*)$	434.221
C_A	-20	1.11981	368.411	1.53682	379.296	1.63198	379.757	$Z_1(T_1^*)$	368.411
	-10	1.18809	384.010	1.62765	390.674	1.69824	390.567	$Z_1(T_1^*)$	384.010
	+10	1.31476	412.780	1.80141	411.679	1.82282	411.014	$Z_3(T_1^*)$	411.014
	+20	1.37410	426.168	1.88513	421.445	1.88157	420.732	$Z_3(T_1^*)$	420.732
p	-20	1.29398	404.494	1.66931	406.189	1.72479	405.896	$Z_3(T_1^*)$	404.494
	-10	1.27285	401.663	1.69182	403.836	1.74342	403.450	$Z_1(T_1^*)$	401.663
	+10	1.23402	395.786	1.74109	399.006	1.77985	398.462	$Z_1(T_1^*)$	395.786
	+20	1.21611	392.750	1.76823	396.525	1.79762	395.923	$Z_1(T_1^*)$	392.750
a	-20	1.40293	346.120	1.92633	340.934	1.91014	340.384	$Z_3(T_1^*)$	340.384
	-10	1.32147	372.867	1.81080	371.508	1.82946	370.898	$Z_3(T_1^*)$	370.898
	+10	1.19412	423.923	1.63576	430.844	1.70413	430.682	$Z_1(T_1^*)$	423.923
	+20	1.14299	448.458	1.56744	459.794	1.65439	460.090	$Z_1(T_1^*)$	448.458
m	-20	1.26871	394.754	1.71542	396.728	1.75925	396.267	$Z_1(T_1^*)$	394.754
	-10	1.26044	396.787	1.71438	399.144	1.75957	398.681	$Z_1(T_1^*)$	396.787
	+10	1.24604	400.671	1.71950	403.624	1.76595	403.136	$Z_1(T_1^*)$	400.671
	+20	1.23984	402.524	1.72598	405.686	1.77218	405.173	$Z_1(T_1^*)$	402.524
k	-20	1.46530	364.548	2.39638	348.934	2.14613	349.976	$Z_3(T_1^*)$	349.976
	-10	1.33769	384.015	1.95273	379.477	1.92372	378.815	$Z_3(T_1^*)$	378.815
	+10	1.19275	410.296	1.57058	418.069	1.64509	418.345	$Z_1(T_1^*)$	410.296
	+20	1.14799	419.561	1.47257	431.129	1.55900	432.275	$Z_1(T_1^*)$	419.561
θ	-20	1.26114	396.444	1.73148	398.526	1.77314	398.008	$Z_1(T_1^*)$	396.444
	-10	1.25699	397.605	1.72349	399.990	1.76740	399.495	$Z_1(T_1^*)$	397.605
	+10	1.24887	399.907	1.70808	402.883	1.75627	402.437	$Z_1(T_1^*)$	399.907
	+20	1.24490	401.049	1.70063	404.313	1.75088	403.891	$Z_1(T_1^*)$	401.049
C_d	-20	1.25873	397.378	1.72995	399.585	1.77243	399.070	$Z_1(T_1^*)$	397.378
	-10	1.25580	398.069	1.72278	400.515	1.76709	400.022	$Z_1(T_1^*)$	398.069
	+10	1.25001	399.447	1.70870	402.366	1.75651	401.918	$Z_1(T_1^*)$	399.447
	+20	1.24714	400.133	1.70179	403.286	1.75128	402.862	$Z_1(T_1^*)$	400.133
C_r	-20	1.25294	398.709	1.71580	401.389	1.76186	400.918	$Z_1(T_1^*)$	398.709
	-10	1.25292	398.734	1.71575	401.416	1.76182	400.945	$Z_1(T_1^*)$	398.734
	+10	1.25287	398.784	1.71564	401.469	1.76174	400.998	$Z_1(T_1^*)$	398.784
	+20	1.25285	398.808	1.71559	41.4960	1.76170	401.025	$Z_1(T_1^*)$	398.808
C	-20	1.25756	391.655	1.72707	393.958	1.77029	393.451	$Z_1(T_1^*)$	391.655
	-10	1.25522	395.207	1.72136	397.701	1.76602	397.212	$Z_1(T_1^*)$	395.207
	+10	1.25058	402.309	1.71009	405.181	1.75756	404.729	$Z_1(T_1^*)$	402.309
	+20	1.24829	405.859	1.70454	408.918	1.75336	408.485	$Z_1(T_1^*)$	405.859
C_h	-20	1.31048	385.626	1.86431	383.516	1.86733	382.803	$Z_3(T_1^*)$	382.803
	-10	1.28078	392.263	1.78541	392.649	1.81281	392.007	$Z_3(T_1^*)$	392.007
	+10	1.22665	405.122	1.65353	410.563	1.71402	409.708	$Z_1(T_1^*)$	405.122
	+20	1.20191	411.36	1.59766	418.145	1.66930	418.231	$Z_1(T_1^*)$	411.360

Table 2
(For example (2)) The effect of changing the parameter (i) while keeping all other parameters unchanged

Parameter (i)	% Change	T_1^*	$Z_1(T_1^*)$	T_2^*	$Z_2(T_2^*)$	T_3^*	$Z_3(T_3^*)$	Optimal Solution	TC
C_p	-20	1.81323	442.892	2.85506	437.557	2.95676	434.985	$Z_3(T_3^*)$	434.985
	-10	1.79610	461.768	2.75217	458.509	2.90722	455.891	$Z_3(T_3^*)$	455.891
	+10	1.76310	499.422	2.59462	499.761	2.81884	497.456	$Z_3(T_3^*)$	497.456
	+20	1.74721	518.201	2.53165	520.188	2.77928	518.130	$Z_3(T_3^*)$	518.130
C_A	-20	1.58531	438.994	2.33311	451.201	2.63309	451.236	$Z_1(T_1^*)$	438.994
	-10	1.68462	460.402	2.50028	465.689	2.75028	464.239	$Z_1(T_1^*)$	460.402
	+10	1.87037	499.791	2.83369	491.970	2.96645	488.723	$Z_3(T_3^*)$	488.723
	+20	1.95811	518.076	2.99996	503.972	3.06568	500.326	$Z_3(T_3^*)$	500.326
p	-20	1.84287	488.940	2.53810	487.481	2.75589	485.854	$Z_3(T_3^*)$	485.854
	-10	1.81007	484.832	2.59860	483.429	2.80883	481.339	$Z_3(T_3^*)$	481.339
	+10	1.75060	476.285	2.74591	474.893	2.91322	471.977	$Z_3(T_3^*)$	471.977
	+20	1.72349	471.860	2.83880	470.358	2.96407	467.139	$Z_3(T_3^*)$	467.139
a	-20	2.00092	421.533	3.08277	407.781	3.11316	404.793	$Z_3(T_3^*)$	404.793
	-10	1.88027	451.679	2.85219	444.004	2.97776	441.028	$Z_3(T_3^*)$	441.028
	+10	1.69341	508.515	2.51545	513.654	2.76063	511.933	$Z_1(T_1^*)$	508.515
	+20	1.61897	535.531	2.38891	547.372	2.67284	546.76	$Z_1(T_1^*)$	535.531
m	-20	1.79006	476.407	2.56167	476.096	2.75873	474.394	$Z_3(T_3^*)$	474.394
	-10	1.78397	478.573	2.60677	477.862	2.80456	475.777	$Z_3(T_3^*)$	475.777
	+10	1.77627	482.525	2.74735	480.188	2.92970	477.175	$Z_3(T_3^*)$	477.175
	+20	1.77455	484.317	2.85462	480.644	3.00880	477.145	$Z_3(T_3^*)$	477.145
k	-20	2.12770	430.249	4.06453	393.610	3.54119	392.950	$Z_3(T_3^*)$	392.950
	-10	1.91876	458.746	3.25081	442.920	3.19904	439.201	$Z_3(T_3^*)$	439.201
	+10	1.68079	497.859	2.34213	506.344	2.58945	506.624	$Z_1(T_1^*)$	497.859
	+20	1.60773	511.782	2.14123	527.412	2.38914	530.605	$Z_1(T_1^*)$	511.782
θ	-20	1.78812	477.607	2.67743	475.658	2.86441	473.100	$Z_3(T_3^*)$	473.100
	-10	1.78371	479.114	2.67205	477.460	2.86273	474.917	$Z_3(T_3^*)$	474.917
	+10	1.77517	482.098	2.66246	481.001	2.86033	478.484	$Z_3(T_3^*)$	478.484
	+20	1.77103	483.575	2.65824	482.742	2.85960	480.235	$Z_3(T_3^*)$	480.235
C_d	-20	1.78769	478.694	2.69617	476.502	2.88205	473.816	$Z_3(T_3^*)$	473.816
	-10	1.78353	479.653	2.68150	477.874	2.87169	475.266	$Z_3(T_3^*)$	475.266
	+10	1.77528	481.567	2.65286	480.601	2.85111	478.154	$Z_3(T_3^*)$	478.154
	+20	1.77120	482.520	2.63887	481.955	2.84091	479.591	$Z_3(T_3^*)$	479.591
C_r	-20	1.78072	480.234	2.67172	478.732	2.8647	476.178	$Z_3(T_3^*)$	476.178
	-10	1.78005	480.423	2.66939	478.987	2.86304	476.445	$Z_3(T_3^*)$	476.445
	+10	1.77873	480.799	2.66475	479.494	2.85971	476.979	$Z_3(T_3^*)$	476.979
	+20	1.77807	480.988	2.66244	479.748	2.85805	477.246	$Z_3(T_3^*)$	477.246
C	-20	1.78602	473.078	2.69027	471.052	2.87790	468.397	$Z_3(T_3^*)$	468.397
	-10	1.78270	476.845	2.6786	475.148	2.86962	472.556	$Z_3(T_3^*)$	472.556
	+10	1.77610	484.376	2.65568	483.329	2.85316	480.866	$Z_3(T_3^*)$	480.866
	+20	1.77283	488.139	2.64444	487.414	2.84498	485.016	$Z_3(T_3^*)$	485.016
C_h	-20	1.86115	462.387	2.98032	452.648	3.06107	449.024	$Z_1(T_1^*)$	462.387
	-10	1.81904	471.592	2.8114	466.244	2.95946	463.054	$Z_3(T_3^*)$	463.054
	+10	1.74200	489.454	2.54265	491.709	2.76771	490.006	$Z_1(T_1^*)$	489.454
	+20	1.70667	498.128	2.43429	503.71	2.67903	502.947	$Z_1(T_1^*)$	498.128

The behavior of the parameters changed with respect to the total average cost is shown graphically in Fig. 10 (for example (1)) and Fig. 11 (for example (2)) and some interesting results drawn from sensitivity analysis are given as follows.

(1) The total average cost increases as the purchasing cost (C_p) increases, which is true in practical situation. As the purchasing cost per item increases, it is obvious to increase the optimal cost of the system.

(2) The total average cost of the system increases with an increase in ordering cost (C_A), which is quite natural as the per order growth of ordering cost implies an increase in total average cost of the system.

(3) When the selling price increases the total average cost of the inventory system decreases. The fact is that due to the higher selling price retailer accumulates more revenue and earns more interest during the delay period.

(4) As the demand parameters (a, m) increases the total average cost of the system increases. The motive is that more demand means more production consequently the total average cost increases.

(5) An increase in production parameter (k) shows that the retailer produces more items therefore the holding cost and deterioration cost, etc. increases as a result the optimal cost of the system increases.

(6) The total average cost decreases as the deterioration rate (θ) and the deterioration cost (C_d) decreases which according to the real situation.

(7) An increase in rework cost per unit item indicates the growth of the total rework cost. To reduce the cost, the production of imperfect items will have to be reduced.

(8) When the production cost (C) and the holding cost (C_h) increases the total average cost of the system increases. The reason is that per unit increase in production and holding costs increases the total production and holding costs therefore the total average cost of the proposed model increases.

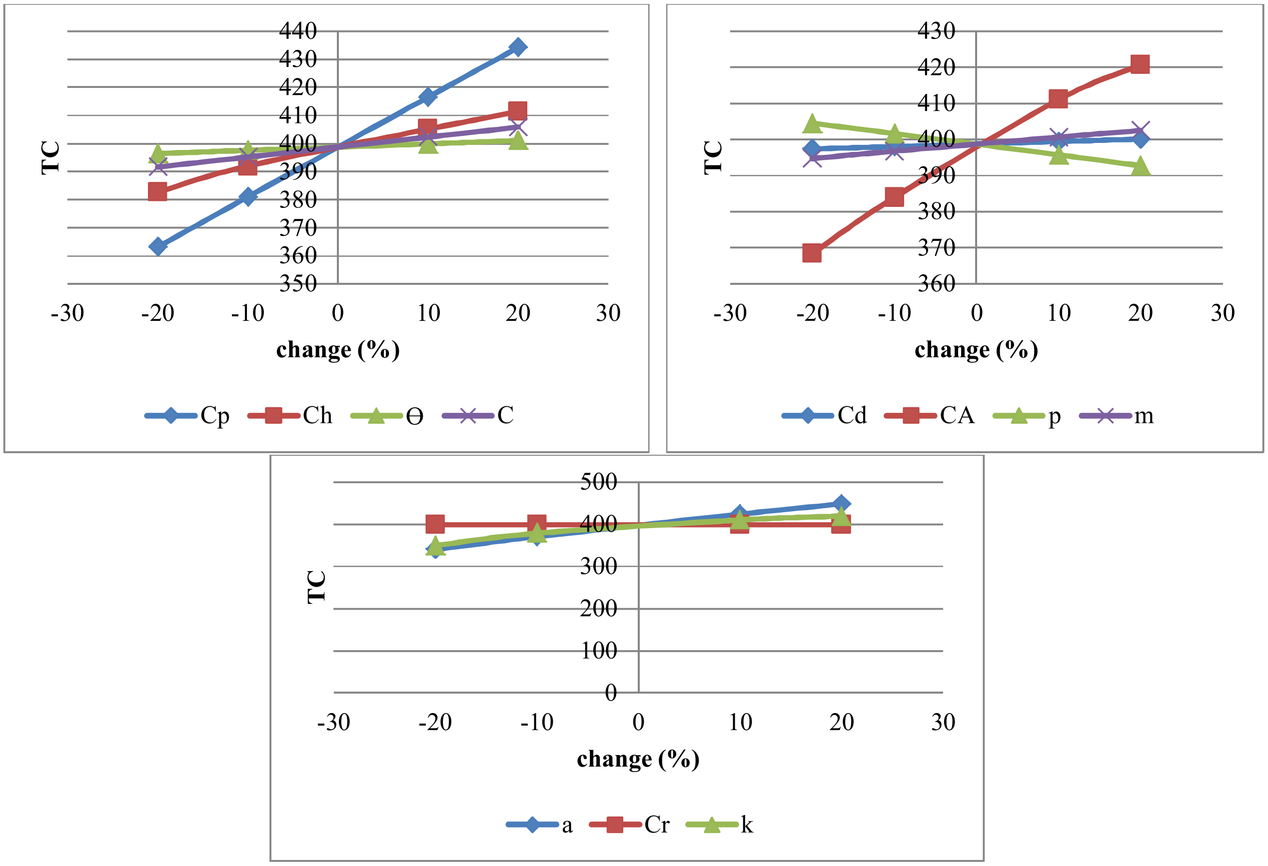

Fig. 10. (For example (1)) The effect of changing parameters on the optimal cost function

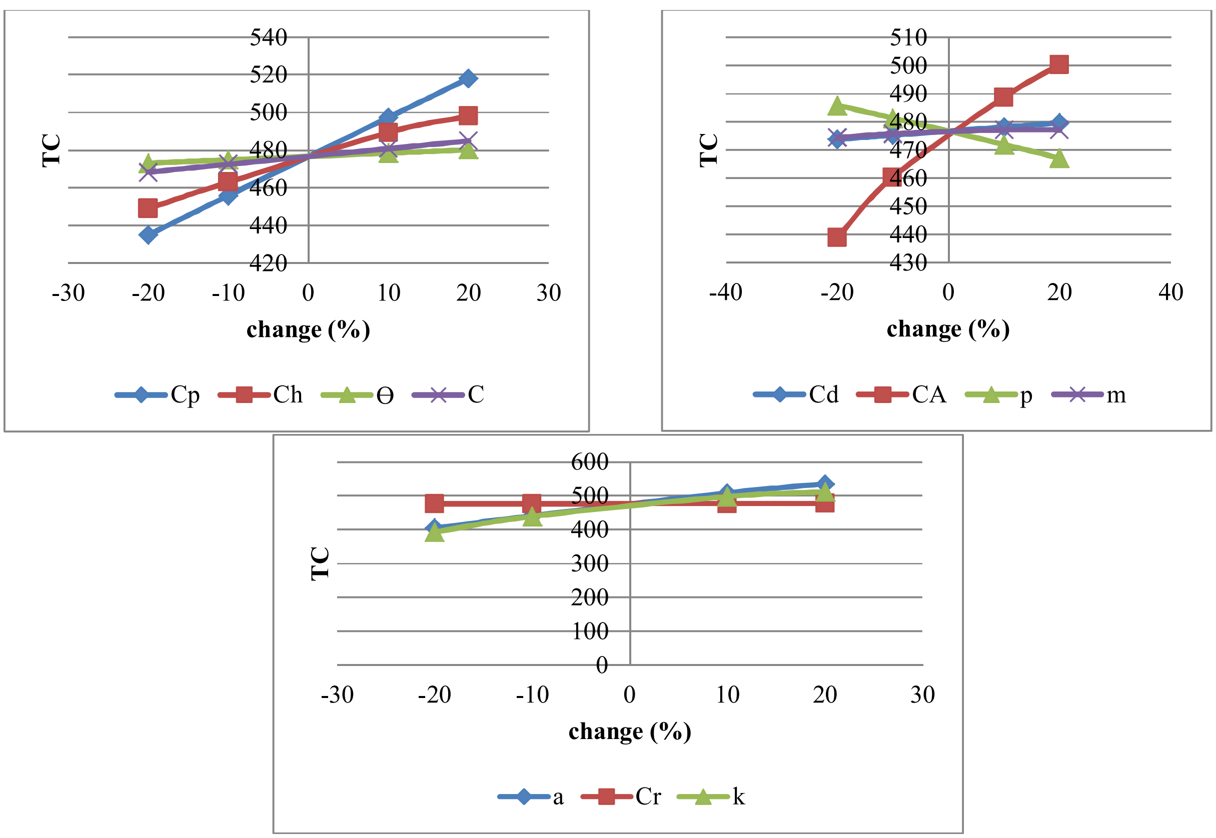

Fig. 11. (For example (2)) The effect of changing parameters on the optimal cost function

6. Conclusion

In this research article, an inventory model for deteriorating items with stock dependent demand rate considering imperfect production and delay in payment scheme has been developed. In this model, two delay periods have been provided by the supplier to attract the retailer. During the delay period an interest was earned on accumulated revenue by the retailer selling his/her commodity. In most of the papers, the examiners have considered the production of the perfect items through different machinery systems. However, in practical situation, due to employment problems, machine breakdowns, the system produces imperfect quality items, which may rework at a cost to make it perfect. In this model, the production of the imperfect items follows Weibull distribution and the production rate depends on the demand factor. An algorithm to determine the optimal policy has also been presented. In addition, sensitivity analysis is performed to examine the effect of parameters. From sensitivity analysis it is observed that the model is enough stable with respect to the changes in system parameters. Further, the model may be generalized by considering shortages and n cycles in a finite planning horizon.

Appendix 1.

The values are given as follows:

$$\phi_1=\frac{ae^{-(m+\theta)t_1}}{(m+\theta)^2}\left\{(k-1)+e^{(m+\theta)T}\right\}-\frac{ak}{(m+\theta)^2}+\frac{a(kt_1-T)}{(m+\theta)}\qquad \phi_2=kat_1$$

$$\phi_3=\left[(R-T+t_1)\left\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\left(e^{-(m+\theta)t_1}-1\right)\right\}+R\left\{a(T-t_1)+\frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma(T-t_1)}{(m+\theta)}\right\}\right.$$
$$\left.-\left\{\frac{aT^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1}-1\right)+\frac{ma}{(m+\theta)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-T\right)+\frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma\left(T^2-t_1^2\right)}{2(m+\theta)}\right\}\right]$$

$$\phi_4=\left[1-e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}\right],\ \phi_5=\frac{d\phi_1}{dT}=\frac{a(k-1)\left(1-e^{-(m+\theta)T}\right)}{(m+\theta)\left[1+(k-1)e^{-(m+\theta)T}\right]},$$

$$\phi_6=\frac{d\phi_3}{dT}=(R-T+t_1)\frac{d\phi_7}{dT}-\frac{(k-1)\phi_7}{\left[e^{(m+\theta)T}+(k-1)\right]}+R\frac{d\phi_8}{dT}-\frac{d\phi_9}{dT}$$

$$\phi_7=\left\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\left(e^{-(m+\theta)t_1}-1\right)\right\}$$

$$\phi_8=\left\{a(T-t_1)+\frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma(T-t_1)}{(m+\theta)}\right\}$$

$$\phi_9=\left\{\frac{aT^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1}-1\right)+\frac{ma}{(m+\theta)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-T\right)\right.$$
$$\left.+\frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma\left(T^2-t_1^2\right)}{2(m+\theta)}\right\}$$

$$\frac{d\phi_7}{dT}=\frac{a}{\left[1+(k-1)e^{-(m+\theta)T}\right]}\left\{1+\frac{m(k-1)}{(m+\theta)}\left(1-e^{-(m+\theta)t_1}\right)\right\}\qquad \frac{d\phi_8}{dT}=\frac{a(k-1)\left\{\theta+me^{-(m+\theta)(t_1-T)}\right\}}{(m+\theta)\left[e^{(m+\theta)T}+(k-1)\right]}$$

$$\frac{d\phi_9}{dT}=\frac{ma\left\{kt_1+(k-1)e^{-(m+\theta)t_1}+\dfrac{e^{-(m+\theta)(t_1-T)}}{(m+\theta)}\right\}}{(m+\theta)\left[1+(k-1)e^{-(m+\theta)T}\right]}+\left\{\frac{a\theta T}{(m+\theta)}-\frac{am}{(m+\theta)^2}\right\}$$

$$\phi_{10}=\frac{d\phi_4}{dT}=\frac{\alpha t_1^{\beta}e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}}{\left[1+(k-1)e^{-(m+\theta)T}\right]}\qquad \phi_{11}=\frac{ka}{\left[1+(k-1)e^{-(m+\theta)T}\right]}$$

$$\phi_{12}=\frac{d\phi_5}{dT}=\frac{ka(k-1)e^{-(m+\theta)T}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}\qquad \phi_{13}=\frac{d\phi_{11}}{dT}=\frac{ka(k-1)(m+\theta)e^{-(m+\theta)T}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}$$

$$\phi_{14}=\frac{d\phi_6}{dT}=\frac{(k-1)(m+\theta)e^{-(m+\theta)T}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}\phi_7-\frac{2(k-1)e^{-(m+\theta)T}}{\left[1+(k-1)e^{-(m+\theta)T}\right]}\frac{d\phi_7}{dT}+(R-T+t_1)\frac{d^2\phi_7}{dT^2}+R\frac{d^2\phi_8}{dT^2}-\frac{d^2\phi_9}{dT^2}$$

$$\frac{d^2\phi_7}{dT^2}=\frac{a(k-1)}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}\left[me^{-(m+\theta)t_1}+e^{-(m+\theta)T}\left\{1+\frac{m(k-1)}{(m+\theta)}\left(1-e^{-(m+\theta)t_1}\right)\right\}\right]$$

$$\frac{d^2\phi_8}{dT^2}=\frac{a(k-1)}{\left[e^{(m+\theta)T}+(k-1)\right]^2}\left[m(k-1)e^{-(m+\theta)(t_1-T)}-e^{(m+\theta)T}\left(\theta+me^{-(m+\theta)(t_1-T)}\right)\right]$$

$$\frac{d^2\phi_9}{dT^2}=\frac{ma(k-1)e^{-(m+\theta)T}\left\{(k-1)e^{-(m+\theta)t_1}+kt_1\right\}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}+\frac{ma(k-1)e^{-(m+\theta)T}\left\{k+(k-1)e^{-(m+\theta)t_1}\right\}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}+\frac{a\theta}{(m+\theta)}$$

$$\phi_{15}=\frac{d\phi_{10}}{dT}=\frac{\alpha t_1^{\beta-1}e^{-\left(\frac{\alpha}{\beta+1}\right)t_1^{\beta+1}}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}\left[(k-1)(m+\theta)t_1e^{-(m+\theta)T}+\beta-\alpha t_1^{\beta+1}\right]$$

$$\phi_{16}=\left[\phi_{12}(C_h+\theta C_d)+\phi_{13}(C_p+C)+\phi_{15}C_rka-\phi_{14}pI_e\right]$$

$$\phi_{17}=2\left[\phi_6pI_e-\phi_5(C_h+\theta C_d)-\phi_{11}(C_p+C)-\phi_{10}C_rka\right]$$

$$\phi_{18}=2\left[C_A+\phi_1(C_h+\theta C_d)+\phi_2(C_p+C)+\phi_4C_rka-\phi_3pI_e\right]$$

Appendix 2.

The values are given as follows:

$$\phi_{19}=\left[t_1\left\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\left(e^{-(m+\theta)t_1}-1\right)\right\}+T\left\{a(T-t_1)+\frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma(T-t_1)}{(m+\theta)}\right\}\right.$$
$$-\left\{\frac{aT^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1}-1\right)+\frac{ma}{(m+\theta)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-T\right)\right.$$
$$\left.\left.+\frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)}-1\right)-\frac{ma(T^2-t_1^2)}{2(m+\theta)}\right\}\right]$$

$$\phi_{20}=\frac{d\phi_{19}}{dT}=\frac{\phi_7}{\left[1+(k-1)e^{-(m+\theta)T}\right]}+t_1\frac{d\phi_7}{dT}+\phi_8+T\frac{d\phi_8}{dT}-\frac{d\phi_9}{dT}$$

$$\phi_{21}=\frac{a}{(m+\theta)}\left[(R-T)-\frac{\left(1-e^{-(m+\theta)(R-T)}\right)}{(m+\theta)}\right]\qquad \phi_{22}=\frac{a\left[e^{-(m+\theta)(R-T)}-1\right]}{(m+\theta)}$$

$$\phi_{23}=\frac{d\phi_{20}}{dT}=\frac{(k-1)e^{-(m+\theta)T}\phi_7}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}+\frac{2}{\left[1+(k-1)e^{-(m+\theta)T}\right]}\frac{d\phi_7}{dT}+t_1\frac{d^2\phi_7}{dT^2}+2\frac{d\phi_8}{dT}+T\frac{d^2\phi_8}{dT^2}-\frac{d^2\phi_9}{dT^2}$$

$$\phi_{24}=\frac{d\phi_{22}}{dT}=ae^{-(m+\theta)(R-T)}$$

$$\phi_{25}=\left[\phi_{12}\left(C_h+\theta C_d\right)+\phi_{13}\left(C_p+C\right)+\phi_{15}C_r ka+\phi_{24}C_p I_{c1}-\phi_{23}pI_e\right]$$

$$\phi_{26}=2\left[\phi_{20}pI_e-\phi_5\left(C_h+\theta C_d\right)-\phi_{11}\left(C_p+C\right)-\phi_{10}C_r ka-\phi_{22}C_p I_{c1}\right]$$

$$\phi_{27}=2\left[C_A+\phi_1\left(C_h+\theta C_d\right)+\phi_2\left(C_p+C\right)+\phi_4 C_r ka+\phi_{21}C_p I_{c1}-\phi_{19}pI_e\right]$$

Appendix 3.

The values are given as follows:

$$\begin{aligned}\phi_{28}=\Bigg[&t_1\left\{at_1+\frac{m(k-1)at_1}{(m+\theta)}+\frac{m(k-1)a}{(m+\theta)^2}\left(e^{-(m+\theta)t_1}-1\right)\right\}+T\left\{a(S-t_1)+\frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\right)-\frac{ma(S-t_1)}{(m+\theta)}\right\}\\&-\left\{\frac{aS^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1}-1\right)+\frac{ma}{(m+\theta)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-Se^{-(m+\theta)(S-T)}\right)\right.\\&\left.+\frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\right)-\frac{ma\left(S^2-t_1^2\right)}{2(m+\theta)}\right\}\Bigg]\end{aligned}$$

$$\phi_{29}=\frac{d\phi_{28}}{dT}=\frac{\phi_7}{\left[1+(k-1)e^{-(m+\theta)T}\right]}+t_1\frac{d\phi_7}{dT}+\phi_{32}+T\frac{d\phi_{32}}{dT}-\frac{d\phi_{33}}{dT}$$

$$\phi_{30}=\frac{a}{(m+\theta)}\left[(S-T)-\frac{\left(1-e^{-(m+\theta)(S-T)}\right)}{(m+\theta)}\right]\qquad \phi_{31}=\frac{d\phi_{30}}{dT}=\frac{a\left[e^{-(m+\theta)(S-T)}-1\right]}{(m+\theta)}$$

$$\phi_{32}=\left\{a(S-t_1)+\frac{ma}{(m+\theta)^2}\left(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\right)-\frac{ma(S-t_1)}{(m+\theta)}\right\}$$

$$\begin{aligned}\phi_{33}=&\left\{\frac{aS^2}{2}+\frac{m(k-1)at_1^2}{2(m+\theta)}+\frac{m(k-1)at_1e^{-(m+\theta)t_1}}{(m+\theta)^2}+\frac{m(k-1)a}{(m+\theta)^3}\left(e^{-(m+\theta)t_1}-1\right)+\frac{ma}{(m+\theta)^2}\left(t_1e^{-(m+\theta)(t_1-T)}-Se^{-(m+\theta)(S-T)}\right)\right.\\&\left.+\frac{ma}{(m+\theta)^3}\left(e^{-(m+\theta)(t_1-T)}-e^{-(m+\theta)(S-T)}\right)-\frac{ma\left(S^2-t_1^2\right)}{2(m+\theta)}\right\}\end{aligned}$$

$$\frac{d\phi_{32}}{dT}=\left[\frac{a\left\{m(k-1)e^{-(m+\theta)t_1}-\theta\right\}}{(m+\theta)\left[1+(k-1)e^{-(m+\theta)T}\right]}-\frac{mae^{-(m+\theta)(S-T)}}{(m+\theta)}\right]$$

$$\frac{d\phi_{33}}{dT}=\frac{ma}{(m+\theta)}\left[\frac{1}{\left[1+(k-1)e^{-(m+\theta)T}\right]}\left\{t_1+\frac{e^{-(m+\theta)(t_1-T)}}{(m+\theta)}+\frac{(k-1)e^{-(m+\theta)t_1}}{(m+\theta)}\right\}-Se^{-(m+\theta)(S-T)}-\frac{e^{-(m+\theta)(S-T)}}{(m+\theta)}\right]$$

$$\frac{d^2\phi_{32}}{dT^2}=\frac{a(k-1)\left[m(k-1)e^{-(m+\theta)(t_1+T)}-\theta e^{-(m+\theta)T}-me^{-(m+\theta)t_1}\right]}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}-mae^{-(m+\theta)(S-T)}$$

$$\frac{d^2\phi_{33}}{dT^2}=\frac{ma}{(m+\theta)}\left[\frac{(k-1)e^{-(m+\theta)T}}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2}\left\{t_1(m+\theta)+e^{-(m+\theta)(t_1-T)}+(k-1)e^{-(m+\theta)t_1}+\frac{e^{(m+\theta)T}}{(k-1)}\right\}-\left(S(m+\theta)+1\right)e^{-(m+\theta)(S-T)}\right]$$

$$\phi_{34} = \frac{d\phi_{29}}{dT} = \frac{(k-1)e^{-(m+\theta)T}\phi_7}{\left[1+(k-1)e^{-(m+\theta)T}\right]^2} + \frac{2}{\left[1+(k-1)e^{-(m+\theta)T}\right]}\frac{d\phi_7}{dT} + t_1\frac{d^2\phi_7}{dT^2} + 2\frac{d\phi_{32}}{dT} + T\frac{d^2\phi_{32}}{dT^2} - \frac{d^2\phi_{33}}{dT^2}$$

$$\phi_{35} = \frac{d\phi_{31}}{dT} = ae^{-(m+\theta)(S-T)}$$

$$\phi_{36} = \left[\phi_{12}\left(C_h + \theta C_d\right) + \phi_{13}\left(C_p + C\right) + \phi_{15}C_r ka + \phi_{35}C_p I_{c2} - \phi_{34} pI_e\right]$$

$$\phi_{37} = 2\left[\phi_{29} pI_e - \phi_5\left(C_h + \theta C_d\right) - \phi_{11}\left(C_p + C\right) - \phi_{10}C_r ka - \phi_{31}C_p I_{c2}\right]$$

$$\phi_{38} = \left[C_A + \phi_1\left(C_h + \theta C_d\right) + \phi_2\left(C_p + C\right) + \phi_4 C_r ka + \phi_{30}C_p I_{c2} - \phi_{28} pI_e\right]$$

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