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VAGUE ALPHA GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper a new concept of vague closed sets called vague α generalized closed sets is introduced . We have constructed some examples which are useful in the theory of Vague α generalized closed sets in topological spaces.

Keywords: Vague topology, vague α generalized closed sets , vague α generalized open sets.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh in 1965.On the otherhand Gau and Buehrer.D.J in 1993 introduced an extension of fuzzy set theory and basic concepts of vague set theory. Biswas in 2006 gave various results in vague set. The theory of fuzzy topology was introduced by Chang in 1968. Mariapresenti and Arockia Rani in 2016, defined the vague generalized alpha closed set in topological spaces. Closed sets are fundamental objects in a topological space. One can define the topology on a set by using either the axioms for the closed sets. In 1970, Levine initiated the study of generalized closed sets. By definition, a subset S of a topological space X is called generalized closed if cl (A)⊆U whenever A ⊆U and U is open. This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets.

2. METHODOLOGY

PRELIMINARIES

A vague set A in the universe of discourse X is characterized by two membership functions given by:

- A true membership function tA : X → [0, 1] and
- A false membership function fA : X → [0, 1],

where tA(x) is lower bound on the grade of membership of x derived from the "evidence for x". fA(x) is a lower bound on the negation of x derived from the "evidence against x" and tA(x) + fA(x) ≤ 1. Thus the grade of membership of x in the vague set A is bounded by a sub interval [tA(x),1 – fA(x)] of [0, 1]. This indicates that if the actual grade of membership μ(x), then tA(x) ≤ μ(x) ≤ fA(x). The vague set A is written as A = {< x, [tA(x), 1–fA(x)] >| x ∈ X} ,

where the interval [tA(x) ,1 – fA(x)] is called the "vague value of x" in A and is denoted by VA(x).

2.1 Definition

Let A and B be two vague sets of the form

A = {< x, [tA(x), 1–fA(x)] >| x ∈ X} and B = {< x, [tB(x), 1–fB(x)] >| x ∈ X}.Then,

a) A ∩ B = {< x, [tA(x) ∧ tB(x)), ((1 − fA(x) ∧ 1 − fB(x)))] > /x ∈ X}

b) A ∪ B = {< x, [tA(x) ∨ tB(x)), ((1 − fA(x) ∨ 1 − fB(x)))] > /x ∈ X}

c) A = B if and only if A ⊆ B and B⊆ A

d) Ac = {< x, [tA(x), 1–fA(x)] >| x ∈ X}

e) A ⊆ B if and only if tA ≤ tB and 1 - fA(x) ≤ 1-fB(x)

We shall use the notation A ={< x, [tA(x), 1–fA(x)] >} instead of {< x, [tA(x), 1–fA(x)] >| x ∈ X} .

2.2 Definition A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

a. 0,1 ∈τ
b. G1 ∩ G2 ∈ τ
c. ∪ Gi ∈ τ for any family { Gi / ∈ J }⊆ τ

In this case the pair (X, τ) is called vague topological space (VTS in short) and vague set in τ is known as vague open set (VOS in short) in X. The complement Ac of vague open set in vague topological space is (X, τ) called vague closed set (VCS in short) in X.

3. ANALYSIS

VAGUE α GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1

A vague set A in a vague topological space X is said to be a vague α Generalized closed set (VαgCS) if VαCl(A) ⊆ U whenever A ⊆ U and U is a vague open set (VOS) in X. The complement C(A) of a VαgCS A is a VαgOS in X .

Theorem 3.2

Let (X, τ) be a vague topological space A ∈ VαgC(X) and foe every B ∈ VS(X) , A⊆ B ⊆ Vα Cl(A) implies B∈ VαgC(X) .

Proof.

Let a vague set B ⊆ U and U be a vague open set in (X, τ) . Then since A ⊆ B , A ⊆ U and A is a VαgCS ,Vαcl(A) .

By hypothesis , B ⊆ Vα cl(A).

Therefore Vα cl(B) ⊆ Vαcl(Vα cl(A)) ⊆ U , Therefore,Vα cl(B) ⊆ U .

Hence B is a VαgCS(X) .

A vague set A in a vague topological space X is said to be a vague α Generalized closed set (VαgCS) if VαCl(A) ⊆ U whenever A ⊆ U and U is a vague open set (VOS) in X. The complement C(A) of a VαgCS A is a VαgOS in X .

Theorem 3.3

The union of any two VαgCSs is a VαgCS in a vague topological space X.

Proof.

Let A and B be any two VαgCSs in a vague topological space X. Let A ⊆ B ⊆ U where U is a vague open set in X.

Then A ⊆ U and B ⊆ U.

Now Vαcl(A ⊆ B) = (A ⊆ B) ⊆ Vcl(VInt(Vcl(A ⊆ B))) ⊆ (A ⊆ B) ⊆ Vcl(Vcl(A ⊆ B)) ⊆ (A ⊆ B) ⊆ Vcl(A ⊆ B) ⊆ Vcl(A ⊆ B) = Vcl(A) ⊆ Vcl(B) ⊆ U ⊆ U = U, by hypothesis. Hence A ⊆ B is a VαgCS in X.

Theorem 3.4

Every Vague alpha closed set A is a VαgCS in X but not converse .

Proof.

Let A ⊆ U where U is a Vague open set in X .

Now Vαcl(A) = A ⊆ Vcl(Vint(Vcl(A))) ⊆ A ⊆ A = A ⊆ Uby hypothesis. Therefore , A is a VαgCS in X .

Theorem 3.5

Every Vague α closed set A is VαCS in X but not converse .

Proof.

Let A ⊆ U , U is a Vague open set in X .

Then , VαCl(A) = A ⊆ Vcl(Vint(Vcl(A))) ⊆ A ⊆ A = A ⊆ U , by hypothesis. Hence A is a VαCS in X .

Theorem 3.6

If A is a vague open set and a VαgCS in (X,τ). Then A is a Vα-CS in (X, τ).

Proof.

Since A ⊆ A and

A is a Vague open set in (X, τ). By hypothesis, Vαcl(A) ⊆ A.

But A ⊆ VαCl(A). Therefore VαCl(A) = A.

Hence A is a Vα-CS in (X,τ) .

4. CONCLUSION

This dissertation is a study of Vague α generalized closed sets in topological spaces. The Vague α generalized closed sets and Vague α generalized open sets in topological spaces and further defined some examples related to open and closed sets .

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