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UPPER AND LOWER SOLUTIONS METHOD FOR PARTIAL DISCONTINUOUS FRACTIONAL DIFFERENTIAL INCLUSIONS WITH NOT INSTANTANEOUS IMPULSES

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Abstract

In this paper, we use the upper and lower solutions method combined with a fixed point theorem for multivalued maps in Banach algebras due to Dhage for investigations of the existence of solutions of a class of discontinuous partial differential inclusions with not instantaneous impulses. Also, we study the existence of extremal solutions under Lipschitz, Carathéodory and certain monotonicity conditions.

Keywords: fractional differential inclusion, left-sided mixed Riemann-Liouville integral, Caputo fractional order derivative, upper solution, lower solution, extremal solution, fixed point, Banach algebras, not instantaneous impulses.

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